7th SPINE MEETING

<u>F. Rogier</u> - J.F. Roussel - D.Volpert
ONERA

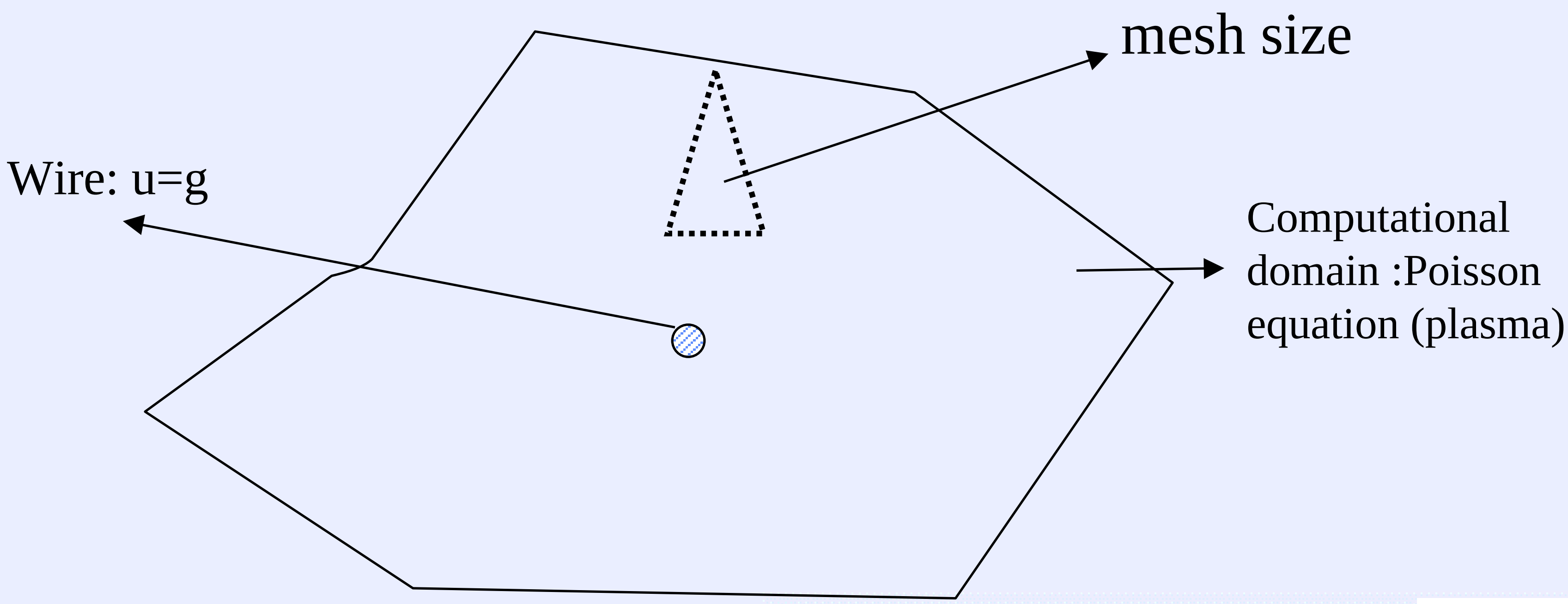
New algorithms for SPIS:
Introduction of wire approximation

SPINE Context

Wire conductors have to take into account : typical radius length ε is very small (millimeter/50m : CLUSTER)

- Mesh the wire => h/ ε >= 1000 => Ncell=Ncell+1000 000 => too large mesh

=> specific treatment of the wire



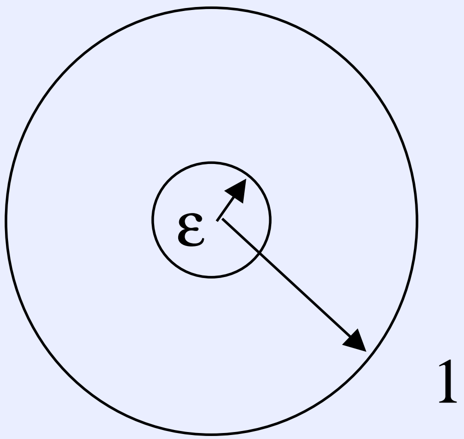
Motivation :example

Let us consider two coaxial conductors :

$$\Delta u = 0$$
$$u(\varepsilon,\theta) = 0$$
$$u(1,\theta) = 1$$

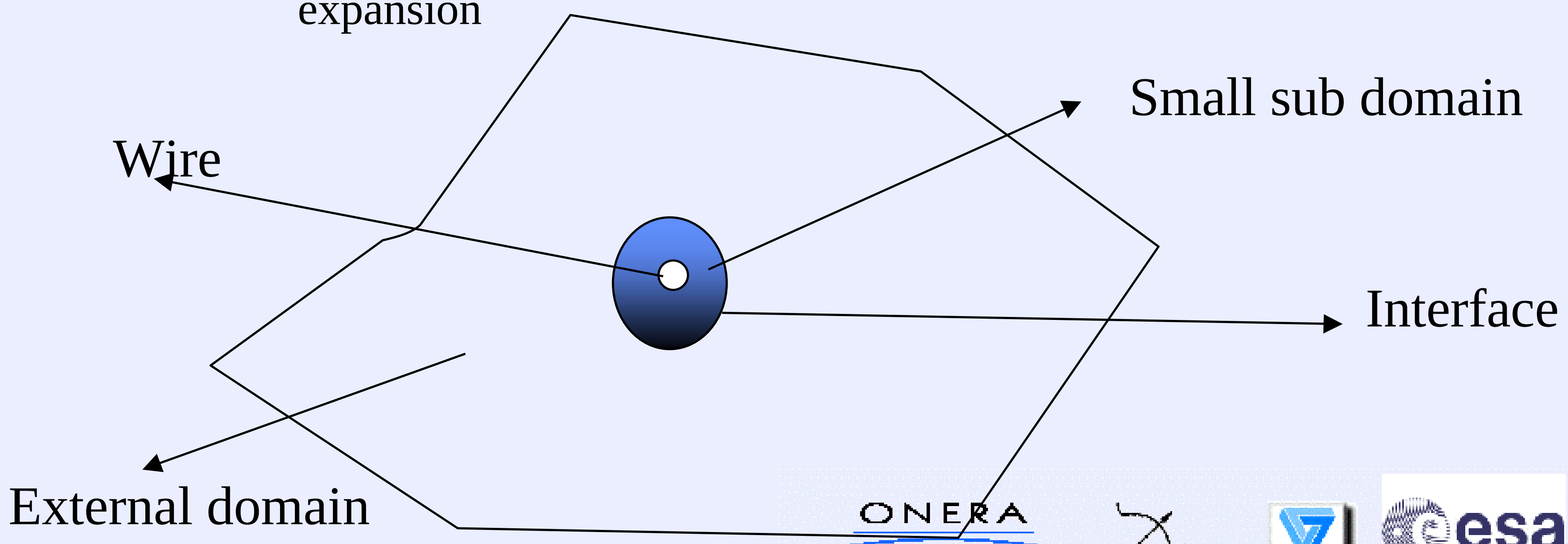
$$u(r,\theta) = 1 - \log(r)/\log(\varepsilon)$$

-The potential is weakly perturbed: O(1/log(ε))
- Asymptotic analysis fails->0



Modelling the wire: 3 steps

- Decompose the domain into a small cylindrical sub domain of a radius “a” containing the wire and the complementary (external sub domain).
- Expand the solution of the Poisson equation in Fourier series into the small sub domain.
- Match the external solution with the analytical expansion



Wire approximation

- Oz axis of the wire (perpendicular to the plane), g(z) is the potential prescribed (on the wire)

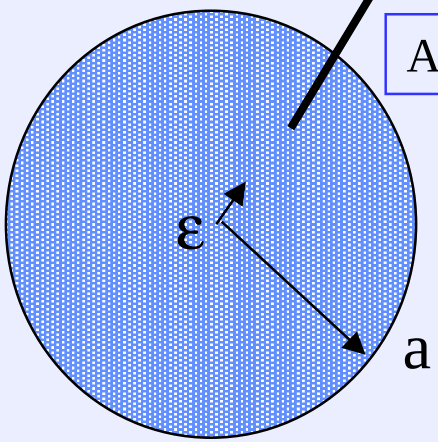
Numerical solution in the external domain

$$\Delta u_{ext} = f$$

$$\frac{\partial u_{ext}}{\partial r}(a,\theta,z) = \frac{\partial u_{fic}}{\partial r}(a,\theta,z)$$

$$u_{fic}(r,\theta,z) \approx (u_{ext}(a,\theta,z))\frac{\log(r/\varepsilon)}{\log(a/\varepsilon)} + g(z)$$

Analytical solution in the small sub domain



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Theoretical Results

- One can approximate the solution by solving the problem into the external domain with Fourier boundary conditions at the interface.

$$\frac{\partial u}{\partial n} + \frac{u}{a\log(a/\varepsilon)} = \frac{g}{a\log(a/\varepsilon)}$$

- Making some assumptions : $\varepsilon / a -> 0$
 $a -> 0$

Error estimate (formally) is O(a, ε/a),

$a = \sqrt{\varepsilon}$ is a good value

Implementation

- Fourier conditions are implemented in SPIS.
- Find the tetrahedrons having one node on the wire and compute the distance to the axis .
- Integration of particle motion -> new numerical scheme (in progress)

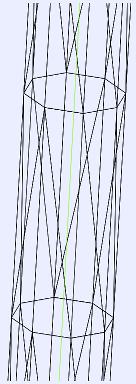
Extension

- Charge space in the fictitious domain can be also taken into account through the Fourier boudary condition (non linear effect have not been investigated yet).
- More accuracy should be obtained by introducing others terms in the Fourier expansion.

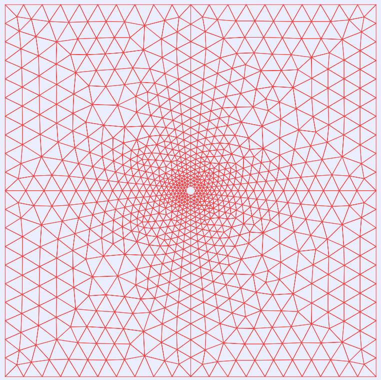
Numerical results

Objective: compare the solution computed from a refined mesh with the wire approximation.

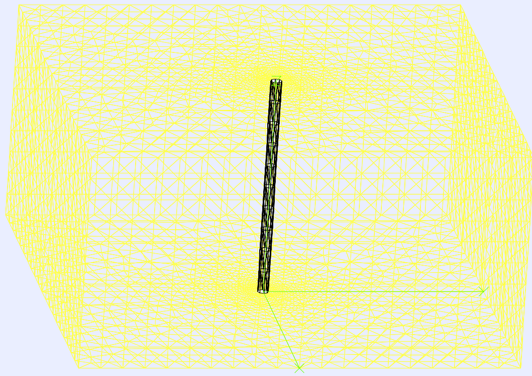
Test case number 1:refined mesh around the wire



Mesh on the wire skin



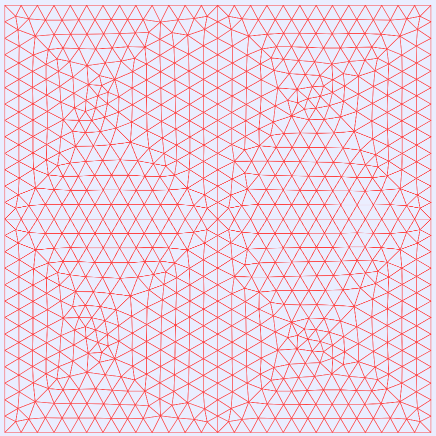
2D cut of the mesh



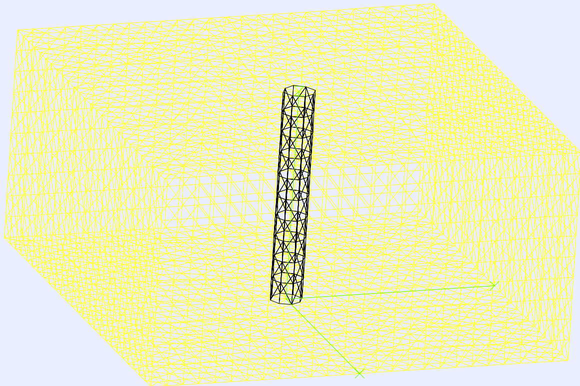
General view:
3D mesh including the wire

Cell number : 170000

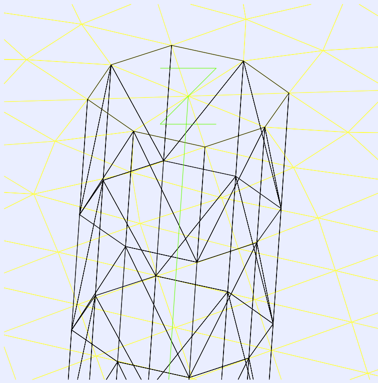
Test case number 2: wire approximation



2D cut of the mesh



General view



Faces close to the wire

Cell number :50 000

Test cases: data

$F(x,y,z) = (x/lx)\sin(2\pi y/ly)\sin(\pi z/lz)$

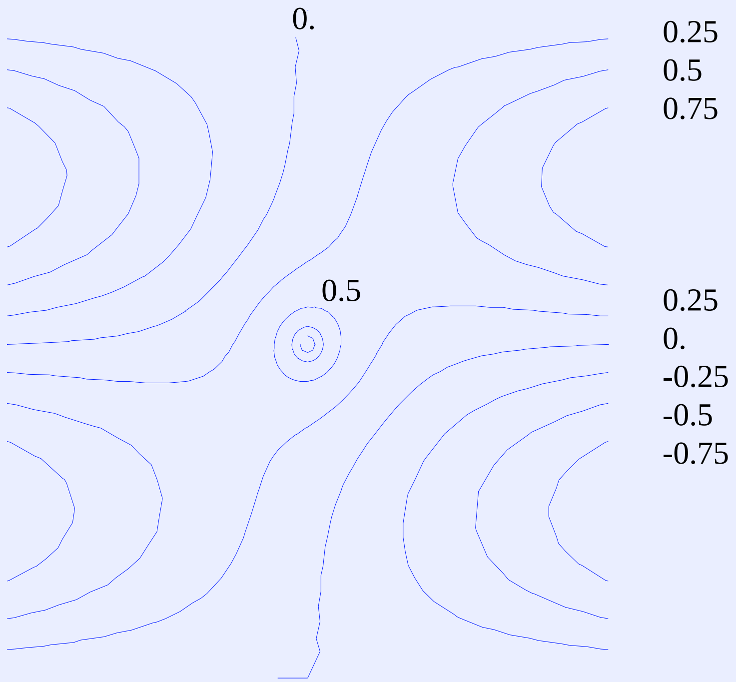
Domain : lx=0.2,ly=0.2,lz=0.1
Wire : $\varepsilon = 0.0001$;

Mesh size : h=0.01->h=0.00005 (test case 1) ;
a=0.01=h (test case 2);

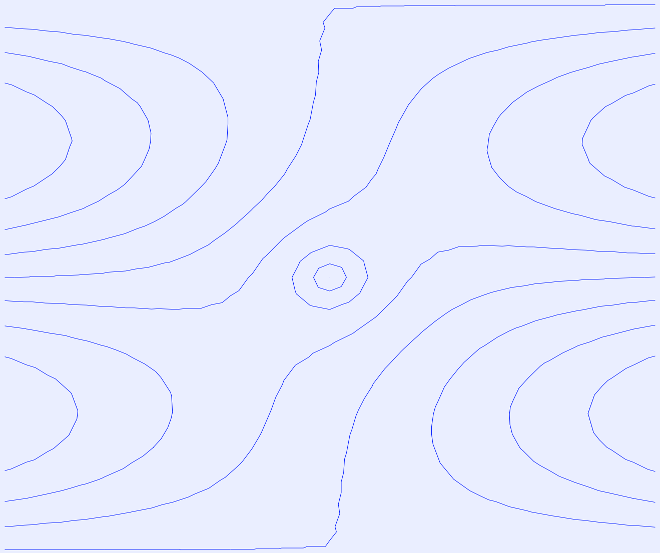
Boundary conditions :
- wire u = 1 (test case 1) ;
-External boundary: u = F;

Test case number 1: numerical results

Iso u in xy plane
z = 0.5

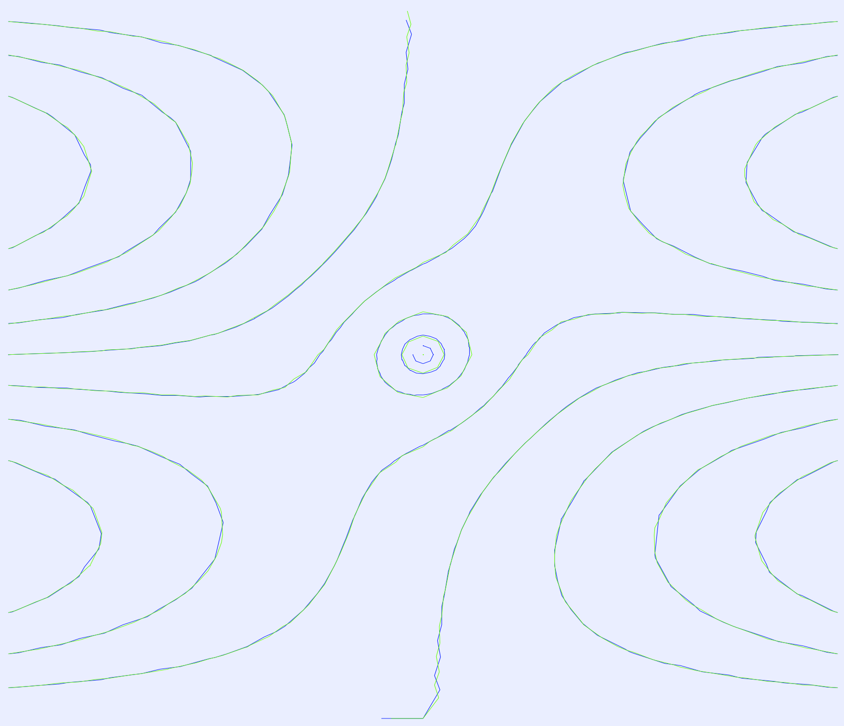


Test case number 2: numerical results



Iso u in xy plane
z = 0.5

Test case 1 and 2



Iso u in xy plane
z = 0.5
Blue = test case 1
Green = test case 2

Test case 1 and 2

Iso u in xy plane
z = 0.5
ZOOM
Blue = test case 1
Green = test case 2

Conclusion

- Numerical results show a good agreement between the wire approximation and the refined computation.
- Need to test the approximation in realistic configuration (CLUSTER).
- Implementation in SPIS is in progress.