Relation between Hall Thruster's Performance and Plume's Curvature

A. Yılmaz

Department of Physics, Science and Letters Faculty,İstanbul Kultur University,

Atakoy Campus, Bakırkoy 34156 Istanbul, Turkey

E-mail a.yilmaz@iku.edu.tr

Abstract

In our study, we began to develop a dynamical model for the Hall thruster's plume by means of family of trajectories related to the exhausted particles. Then, we used the dynamical equations derived for our model plume to obtain equations for the radii of thruster, the thrust, and the trajectory's curvature. After that, the equations found helped us to make connections between the curvature of the plume model and the Hall thruster performance parameters.

1. Introduction

Hall thrusters hold considerable advantage over chemical thrusters and other kinds of electrical propulsion devices, except that the plume of the Hall thruster tends to be wide. An ongoing objective in Hall-thruster research is to narrow this plume. However, in-flight experience data characterizing Hall thruster plumes in the space environment, without the presence of facility artifacts, is necessary to accurately predict plume effects. This is useful to optimize thruster location in the satellite in order to prevent damage from the plume.Plume interaction with the spacecraft remains an important issue despite the considerable flight heritage of Hall thrusters. Possible plasma spacecraft interactions are contamination, supttering and erosion of spacecraft surfaces, degradation of solar arrays, and influence on communication

The development of diagnostic techniques to characterize the plasma inside and outside the thrusters; and the development of simulation and modelling able to describe characteristics and evaluate the thrusters' performances will in turn increase physical understanding of the Hall thruster plume.The validation of these experimental results will establish a database of measurements to support ongoing computational models and physical models by means of analyses which include performing a consistency study between current density and plasma potential and between electron density and plasma potential as well as by a comparison of the experimental results to the self-similar model (Fruchtman et al.2001) .

2. Modeling

If we assume the Hall thruster's plume to be composed of symmetric co-axial surfaces represented by

$$\rho^2 = x^2 + y^2 = \left(\alpha z + \frac{\beta}{\gamma(z-2)^3 + \delta}\right)^2 \qquad (1)$$

then, in terms of inner radius, ρ_{ch}^{i} and outer radii, ρ_{ch}^{o} of the thruster's chamber, the radial distance in the exit plane of the thruster's chamber is given by

$$\rho_{ch} = \left(\frac{\beta}{\delta - 8\gamma}\right) \qquad (2)$$

where $\rho_{ch}^{i} \leq \rho_{ch} \leq \rho_{ch}^{o}$ and $z = 0$. Also, the values of model parameters $\alpha, \beta\ \gamma, \delta$ are choosen as $\alpha = 10^{-8}$, $\beta = (\delta - 8\gamma)\rho_{ch}$, $\gamma = 0.05$, $\delta = 1.5$ to be consistent of experimental and observational data (Yilmaz and Hacibekiroglu, 2008). Hence, each co-axial surface composing the plume given by Eqn.(1) which is also made of symmetric curves traced out by particle trajectories, is determined by rotating one of the curves around the z-axis as a symmetry axis.

We consider the curves representing the particle trajectories in the cross-section of model plume in yz-plane (see Fig.1), i.e.,

$$y(z) = \pm\left(\alpha z + \frac{\beta}{\gamma(z-2)^3 + \delta}\right) \qquad (3)$$

and chose the exhaust velocity of each particle as $\vec{v} = \vec{v}_{ex} = v_y\hat{e}_2 + v_z\hat{e}_3$, amd in turn, $v = v_{ex} = \sqrt{1 + \left(\frac{dy}{dz}\right)^2}\, v_z$.

Consequently,the magnitude of the force exerted on each particle along its trajectory in the plume is determined:

$$F_{ex} = m \frac{\left| \frac{6\gamma}{(\delta - 8\gamma)} \left(\frac{(x-2)\left[2\gamma(z-2)^3 - \delta\right]}{\left[\gamma(z-2)^3 + \delta\right]^3} \right) \rho_{ch} \right|}{\left| \left[1 + \left(\alpha - \frac{3\gamma}{(\delta - 8\gamma)} \left(\frac{(z-2)}{\gamma(z-2)^3 + \delta} \right)^2 \rho_{ch} \right)^2 \right]^{\frac{3}{2}} \right|} v_{ex}^2 \qquad (4)$$

where

$$a_{ex}=\frac{F}{m}=\sqrt{\left(v_z^2\frac{d^2y}{dz^2}+\frac{dy}{dz}a_z\right)^2+a_z^2} \text{ and } a_y=v_z\frac{dv_z}{dz}\frac{dy}{dz}+v_z{}^2\frac{d^2y}{dz^2}; \quad a_z=\frac{dv_z}{dt}=v_z\frac{dv_z}{dz}.$$

On the other hand, since trajectory of each particle will trace out a curve C given explicitly as y=f(z) in the yz-plane, we can parametrized each curve related to the pulme model simply as $c(t)=(y(t),t)$ where $\dot{z}=v_z=1$, $\ddot{z}=a_z=0$, and $\dot{y}=v_z\frac{dy}{dz}$. As a result, the curvature of the particle's trajectory is obtained as:

$$\kappa=\frac{\left|\frac{d^2y}{dz^2}\right|}{\left|\left[1+\left(\frac{dy}{dz}\right)^2\right]^{\frac{3}{2}}\right|} \tag{5}$$

So, taking the time t as the parameter provides a natural parametrization for C and the instanteneous direction of motion is given by the unit tangent vector and the curvature measures how fast this vector rotates.

Also, we show that the force exerted on invidual particle composed the plume, F_{ex} can be expressed in term of the curvature, κ:

$$F_{ex}=\left(m_i v_{ex}^2\right)\kappa \tag{6}$$

3. Results

We plot the radial components of F_{ex} vs ρ_{ch} graph, and the axial components of F_{ex} vs z in a range of $0<z<L_0$ where $F_{ex}=0$ (see Fig.2), and plotting the radial components of F_{ex} vs ρ_{ch} graph, and the axial components of F_{ex} vs ρ_{ch} graph in a $\rho_{ch}^i \le \rho_{ch} \le \rho_{ch}^o$, at $z=0$ (see Fig.3) for a SPT 100 and for a miniaturized Hall thruster of outer radius 1.3 cm. Accordingly, the thrust, T, is contributed by only by the exhausted particle's population on the area of thruster's exit, i.e.,

$$T=n_{ex}^{A_{ex}}2\pi\int_{\rho_{ch}^i}^{\rho_{ch}^o}d\rho_{ch}\,\rho_{ch}\,F_{ex}(\rho_{ch},0) \tag{7}$$

Here, the volume number density of the exhausted particles during each thrust iis defined as

$$n_{ex}=\frac{N_{ex}}{V_{ex}} \text{ where } V_{ex}=A_{ex}L_0=\pi\left(\rho_{ch}^{0\,2}-\rho_{ch}^{i\,2}\right)L_0 \text{ or } n_{ex}=\frac{n_{ex}^{A_{ex}}}{L_0} \tag{8}$$

4. Conclusion

We relate the curvature, κ to the thruster's perfomance parameters by subtituting of Eqn.(8) and Eqn.(9) into Eqn.(10) and the corresponding characteristic equations (Garrgues et al, 2001) :

a) The thrust T: $\vec{T}=\dot{m}_i\vec{v}_i$ where $\dot{m}_i$ is the ion flow rate and $\vec{v}_{ex}$ is the mean value of the exhausted ion speed in the direction of the thrust.

b) The specific impulse I_{sp} defined by $I_{sp}=\frac{T}{\dot{m}g}$ where g is Earth's gravitational acceleration at sea level and $\dot{m}$ is the propellant mass flow rate.

c)The efficiency η which is defined by $\eta=\frac{T^2/2\dot{m}}{P_d}$ where P_d is the electric input power known as system power and P_T is the thrust power defined as $P_T=\frac{T^2}{2\dot{m}}$.

Numerically dynamical plume model gives the particle density contributing to thrust as

- for SPT100 with $\rho_{ch}^i=2.8cm$, $\rho_{ch}^o=5cm$, $\dot{m}=5.6mg/s$, $V_d=300V$, and $T=84mN$ $n_{ex}=1.68\times10^{17}m^{-3}$, and
- for miniaturized Hall thruster with $\rho_{ch}^i=0.7cm$, $\rho_{ch}^o=1.3cm$, $\dot{m}=0.7mg/s$, $V_d=300V$, and $T=10mN$, $n_{ex}=5.0\times10^{16}m^{-3}$ (Smirnov et al., 2002) .

To conclude, our dynamical model can be validatied by applying to the results from the experimental data in space or laboratory on Earth.

Acknowlegments

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References

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Figures

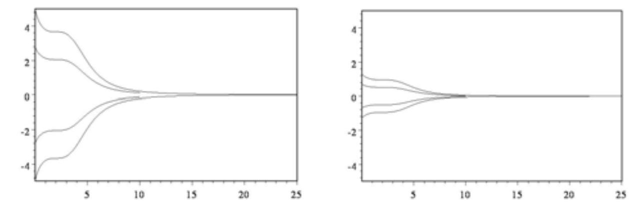

Figure 1 The cross-section of model plume in yz-plane, two symetric trajectory related to inner and outer radii of thruster axis 10 cm away distance along the thruster's axis direction for **a)** a SPT 100 and for **b)** miniaturized Hall thruster of outer radius 1.3 cm.

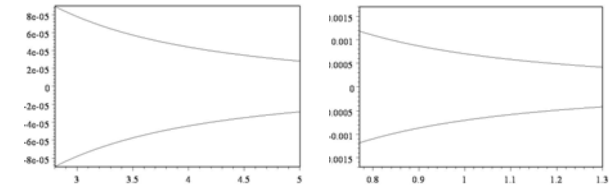

Figure 2 Curvature along thruster's radius direction **a)** a SPT 100 and for **b)** miniaturized Hall thruster of outer radius 1.3 cm.

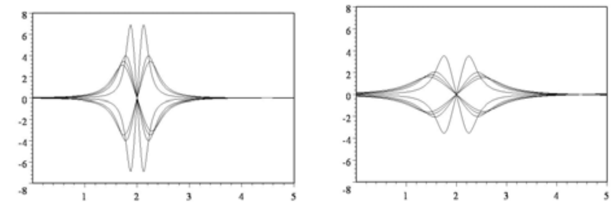

Figure 3 Forces along thruster axial direction for different radii of thruster's channel **a)** a SPT 100 and for **b)** miniaturized Hall thruster of outer radius 1.3 cm.