BRUSHFIRE ARC DISCHARGE MODEL*

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SUMMARY

A 1-dimensional arc discharge model incorporating a brushfire-type propagation of a discharge wavefront has been investigated. A set of equations somewhat similar to those leading to the diffusion equation have been developed which include electrical, thermal, and plasma parameters. The solutions of these equations are shown, under simplifying assumptions, to be consistent with a propagating brushfire wavefront. Voltage, current, plasma density, temperature, and resistivity profiles are obtained.

Mechanical forces, magnetic and electrostatic, are considered in evaluating the flashover to blowout current ratio, G', for arc discharges with the brushfire parameters developed in the model. This ratio is an important factor in determining the electromagnetic interference (EMI) impact of arc discharges on spacecraft electrical subsystems. The conclusion of the analysis is that electrostatic forces are much more important than magnetic forces. The magnitude of the G' factor obtained, 58.5 percent, is within the range of those obtained by experimental means. Improvements in the analytical model as well as in the experimental approach are recommended.

INTRODUCTION

The problem of characterizing dielectric surface arc discharges due to spacecraft charging has been approached mainly by experimental means in the past because of the lack of an analytical model. A number of recent papers have presented analytical approaches to the problem.(1,2) The work presented here is a continued development of the concept of a brushfire propagation model developed by J. M. Sellen Jr. and the author.(3,4)

From the viewpoint of the implications of arc discharges on the immunity of spacecraft to the EMI generated, the question of where the arc discharge currents flow is a critical factor. This problem has been formulated by defining a factor, G', which is defined as the ratio of the blowout to flashover currents. The flashover component is viewed as that which flows essentially from the dielectric surface through a breakdown region, perhaps an edge with high electric fields, directly back to the metallized backing of the dielectric surface. Flashover currents, because their geometrical extent is limited, are not expected to be a major source of spacecraft EMI. Blowout currents, on the other hand, may have a large impact on electrical subsystems because they result in replacement currents flowing through the spacecraft structure which must be of a magnitude equal to the blown off electron current. The density of replacement current flowing in the spacecraft

*This work was supported under NASA Contract NAS3-21961.

structure is highly dependent on the location of the arcing source and on the particular configuration of the spacecraft. An arc on a boom mounted object, for example, may result in boom currents which couple very well into cabling along the boom. A spacecraft body-mounted source, on the other hand, may be so well grounded and shielded that only currents very close to the source are of sufficient magnitude to be of concern. Thus, the determination of a representative value of G' and its dependence on the size of the arcing source and any other parameters is of prime concern for spacecraft design. Any analytical arc discharge model should provide results that are consistent with experimental data. In addition, however, the work presented here predicts facets of the experimental approach, such as the spatial distribution of blowout currents and the dependence of G' on the sample grounding impedance, which were not adequately considered previously.

ARC DISCHARGE OVERVIEW

The brushfire propagation model addresses only the latter portion of the evolutionary processes involved in an arc discharge. The scenario would be as follows:

1. Differential chargeup by the environmental plasma and solar ultraviolet radiation
2. Edge breakdown at a weak point
3. Surface breakdown
   - o High field emission
   - o Avalanching processes
4. Brushfire propagation
   - o Blowout and flashover currents, G'
   - o Dependence on spacecraft potential
   - o Limiting mechanisms on propagation

The question of how external dielectric surfaces charge up differentially with respect to the grounded underlying vacuum deposited aluminum (VDA) or to structural metal is a complex problem which is not addressed here. Generally, the most hazardous situation exists when a dielectric surface is charged negatively with respect to the underlying metals by an excess of impinging electrons over positive ions. This is because with a reverse polarity, i.e., when the metals are negative and the dielectric surface is more positive because of photoemission or secondary emission, a field emission/secondary electron avalanche process tends to limit the magnitude of the differential potential to below 1000 V.

For the purpose at hand of developing an arc discharge model, the chargeup process is imporant in that negative chargeup potentials of 5 kV to

20 kV have been measured experimentally. The other important feature of chargeup for our present purpose is that theory and experimental evidence[5] indicate that significant densities of electrons may be buried at depths of the order of 1 micron below the surface at the time of the discharge. This feature of buried electronic charge should also exist on dielectric surfaces which have no net surface charge because of photoemission or secondary emission. In fact, the buried charge should be somewhat deeper and more dense since retarding potentials are not present.

Dielectric breakdown due to high differential voltage stresses generally occurs for electric fields in the range of 10^5 to 10^6 V/cm at the edges of thin (~50 microns or 0.005 cm) insulating sheets. Punch-through far from the edges occurs with fields of the order of 10^7 V/cm. In practice, even punch-throughs probably occur at weak points where slight imperfections or irregularities exist in the material. Edges consist of exaggerated irregularities because they are created by slicing with a knife edge or by punching with stitching needles, and thus, are subject to high field emission and avalance breakdown in a manner similar to that which will be discussed for surface breakdown. The similarity to surface breakdowns probably goes even further in that this type of breakdown is associated with surface and off-surface processes rather than those within the bulk of the material.

The net effect of an edge breakdown is that the potential of the surface near the edge goes to nearly 0 V, assuming that the thin dielectric is over a conducting plate which is at voltage reference, 0 V. Taking a single ionized particle of atomic weight 16 (oxygen) as being typical, the velocity associated with a 10 kV voltage drop is $3.5 \cdot 10^5$ m/s. Starting at zero velocity, the time for such an ion to traverse the 2 mils or 50 micron thickness of the dielectric is 0.3 ns. This order of magnitude time span, a fraction of a ns, is much shorter than the tens to hundreds of ns duration of vacuum dielectric surface arcs.

Assuming that a 2-mil thick sheet of Kapton, $\varepsilon_r = 3$, breaks down at 10 kV over a semicircular area with a radius equal to its thickness, the capacitance is 52 pf/cm^2 or $2 \cdot 10^{-3}$ pf, and the charge stored is $2 \cdot 10^{-11}$ Coulomb. Assuming that all of this charge is dissipated in 0.3 ns, the corresponding current would be 0.068 A. Thus, the current, charge, time span, and energy (~10^{-7} joule) involved in the initial edge breakdown are quite small and negligible compared to those in the events that follow. The main effect of the initial edge breakdown is to create a plasma cloud and a surface electric field which initiates a subsequent surface dischage.

Dielectric surface breakdown has been reported to occur more readily, at 10^4 to 10^5 V/cm surface electric fields, than breakdown in the bulk of dielectric materials. The surface breakdown fields are expected to be highly dependent on surface conditions such as cleanliness, smoothness and absorbed gases.

BRUSHFIRE PROPAGATION MODEL

The experimentally observed "wipeoff" of charge over many hundreds of cm^2, and possibly greater areas of dielectric surface, requires either some mechanism for propagation of an initial surface breakdown in a brushfire mode or that somehow all of the participating charge release occurs simultaneously over a large area. The propagation mode seems more plausible and is discussed further here. The source of discharging energy, the stored charge per unit area, is depleted, and the discharge must be fed by a forward propagation of the brushfire periphery into the still-charged regions of the dielectric. To discuss the brushfire propagation process, some of the basic equations are presented first. Then, a simplistic piecemeal solution of various aspects of the problem is presented to provide an insight into the quantitative aspects of the problem. Even the basic relations such as those for ablation and ionization are not developed from first principles, but rather, are taken from existing experimental data and theoretical work found in the literature. Figure 1 provides an overview of the brushfire propagation analysis.

The basic equations to be satisfied for the brushfire propagation problem are:

$$\frac{\partial V}{\partial t} = -\frac{1}{C}\frac{\partial J_s}{\partial x} \quad \text{and} \quad J_s = -\frac{1}{\rho_s}\frac{\partial V}{\partial x} \qquad (1,2)$$

where the potential, V, and surface current density, J_s, are functions of horizontal distance, x, and time, t. The two other parameters of this 1-dimensional formulation are the capacitance per unit area, C, which is 52 pf/cm^2 for a 2-mil thick dielectric with a dielectric constant of 3, and the surface resistivity, ρ_s (ohms-per-square), of the plasma sheet that conducts the arc discharge current, J_s. The geometry of the problem is shown in Figure 2. The initial voltage, -5 kV, was selected to give a 10^6 V/cm electric field bulk breakdown for the 2-mil dielectric thickness. A final voltage of -2.5 kV was assumed on the basis that about 50 percent of the initial voltage has been observed experimentally to remain after the discharge. As an initial guess, the voltage is assumed to decrease linearly with distance providing an electric field of 10^4 V/cm. The voltage gradient region is therefore 0.25 cm long. Combining equations (1) and (2) to eliminate J_s gives

$$\frac{\partial V}{\partial t} = \frac{1}{C\rho_s}\frac{\partial^2 V}{\partial x^2} \qquad (3)$$

This would be the diffusion equation with the diffusion coefficient, D:

$$\frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial x^2} \quad \text{where } D = \frac{1}{C\rho_s}$$

except that ρ_s is not a constant in our problem. This is fortunate because the diffusion equation does not lead to a propagating mode with a constant velocity.

The plasma resistivity, ρ, and surface resistivity, ρ_s, are functions of the temperature, T:[6]

$$\rho = \frac{K}{T^{3/2}} \text{ ohm - cm, where } K = 0.03 \text{ ohm-cm-ev}^{3/2} \qquad (4a)$$

$$\rho_s = \rho/d = \frac{K}{d} T^{-3/2} \text{ ohms} \qquad (4b)$$

where d is the thickness of the plasma sheet. It is of interest to note that ρ is independent of the density of the plasma particles.

T is governed by a set of equations similar to those for V:

$$\frac{\partial T}{\partial t} = -\frac{1}{cM}\frac{\partial H}{\partial x}, \quad H = -\frac{1}{R}\frac{\partial T}{\partial x} \qquad (5,6)$$

where H is the heat flux, c is the specific heat, M is the mass density, and R is the thermal resistivity. For our problem here we neglect thermal conductivity, because of the short time spans involved, and assume that R is infinite. The rate of heat energy deposition in an incremental distance, dx, in equation (5) is the power density, P_s:

$$-\frac{\partial H}{\partial x} = P_s = -J_s \frac{\partial V}{\partial x} \text{ watts/cm}^2 \qquad (7)$$

The specific heat, c, is obtained using the gas constant, R, by assuming that the plasma consists of neutrals, ions and electrons, each with 3 degrees of freedom.

$$c_m = \frac{1}{2} \cdot 9R = 4.5R = 4.5 \cdot 8.314 = 37.41 \text{ joule/(deg-mole)} \qquad (8a)$$

Assuming the dielectric material has a molecular weight, G_m, of 16, c is given by:

$$c = c_m/G_m = 2.34 \text{ joule/(deg-gram)} = 2.71 \cdot 10^4 \text{ joule/(ev-gram)} \qquad (8b)$$

where c_m is defined as the specific heat per mole and G_m is defined as the mass density per mole.

The mass density, M, to be used in equation (5) is composed of two components, M_a, due to ablation because of the power dissipation, P_s, and M_0 which is due to the initial field emission electrons:

$$M = M_a + M_o \text{ grams/cm}^2 \tag{9}$$

The ablated mass density, M_a, is assumed to be proportional to the time-integrated power density, P_s:

$$M_a = \int g\, P_s\, dt \text{ grams/cm}^2 \tag{10}$$

The proportionality constant, g, is taken from the pulsed plasma thruster technology data.[7]

$$g = 8.32 \cdot 10^{-6} \text{ grams/joule}$$

We view ablation as being due to "pounding" of the surface by ions which are accelerated by the electric field due to the electrons which have been stored (buried) by the basic spacecraft charging process.

M_o is not due to heating in the thermal sense but rather is due to collisions between the initial electrons, that are emitted or "pulled-out" by high field emission at localized regions of high electric field, and the dielectric surface atoms. The high field emission current density, J, is described in terms of the electric field, E, by:[8]

$$J = 6.5 \cdot 10^{-7}\, E^2 e^{-6.5 \cdot 10^9}$$

According to this equation, J has a nearly step-function increase at

$$E = 6.5 \cdot 10^9 \text{ volt/meter} = 6.5 \cdot 10^7 \text{ V/cm}$$

Experimentally observed threshold electric field intensity of 10^4 V/cm, nearly four orders of magnitude less, must be due to the fact that localized regions of high electric fields exist on a sufficiently small microscopic scale.

M_o may be evaluated by equating the energy gained by these field-emitted electrons to an initial temperature, T_i:

$$k\, \Delta T_i = e\Delta V = eE_b\, \Delta\lambda$$

where k is the Boltzmann constant and e is the electronic charge. We take the characteristic distance, λ, to be the Debye shielding distance:

$$\lambda = 6.9 \sqrt{\frac{T_i}{n}}$$

where T_i is the temperature in °K, and n is the plasma density in number/cc. E_b is the surface breakdown electric field of 10^4 V/cm. These equations may be integrated to give:

$$T_i = \frac{A^2}{n+n_o} \text{ °K, where } A^2 = \left(\frac{6.9eE_b}{2k}\right)^2 = 1.602 \cdot 10^{17}$$

$$T_i = \frac{1.381 \cdot 10^{13}}{n + n_o} \text{ ev where n and } n_o \text{ are in particles/cm}^3 \qquad (11)$$

The constant of integration, n_o, has been introduced approximately in the form of additional number density where T_i varies inversely as the total density, by taking T_i as 2500 ev when n is zero. Recall, that n is the number density due to ablation.

This density, n, is evaluated from the ablated mass density, M_a, by

$$n = 6.02 \cdot 10^{23} \frac{\text{molecules}}{\text{mole}} \left(\frac{1 \text{ mole}}{16 \text{ grams}}\right) M_a \frac{\text{grams}}{\text{cm}^2} \cdot \frac{1}{d \text{ cm}} =$$

$$3.76 \cdot 10^{22} \frac{M_a}{d} \frac{\text{molecules}}{\text{cm}^3}$$

The parameter, d, is the thickness of the plasma film or sheet and is assumed to be 1 percent of the voltage gradient region of 0.0025 cm. The number density, n_o, is

$$n_o = \frac{1.38 \cdot 10^{13}}{2500} = 5.523 \cdot 10^9 \text{ particles/cm}^3 \qquad (12a)$$

The corresponding mass density, M_o, is:

$$M_o = n_o d \cdot \frac{16}{6.02 \cdot 10^{23}} = 3.67 \cdot 10^{-16} \text{ grams/cm}^2 \qquad (12b)$$

SIMPLIFIED ANALYSIS

The simultaneous solution of all of the equations presented up to now is rather complex and requires a computerized solution.

Here, some quantitative feeling for the results is obtained by a piecemeal approach with simplifying assumptions.

The first assumption is that there is a solution in which a constant brushfire propagation velocity, v_b, is appropriate. With this assumption, time variables may be replaced with space variables:

$$x = v_b t; \quad \frac{\partial f}{\partial t} = v_b \frac{\partial f}{\partial x} \tag{13}$$

Equations (1) and (2) may then be integrated to give:

$$J_s = Cv_b \, (V_m - V), \text{ and} \tag{14}$$

$$V = V_m \, (1 - \varepsilon^{-f(x)}), \text{ where } f(x) = Cv_b \int_x^{\ell} \rho_s \, d_x \tag{15}$$

where V_m is the maximum voltage change (2500 volts), and V is the voltage at any point x in the voltage gradient region. For this part of the analysis the zero reference voltage is taken to be the potential at the bottom of the voltage falloff region; i.e., the V = 0 at $x = \ell$.

A further simplification of the problem is obtained by assuming that the voltage profile is known, a linear dropoff to a V_{final} of zero as shown in Figure 2. Temperatures, resistivities, particle densities, current densities as well as a new voltage profile can then be calculated. Consistency of the new voltage profile with the assumed profile will put constraints on the possible values of the parameters involved.

The assumed voltage profile is given by

$$V = V_m \, (1 - \frac{x}{\ell}) = V_m - E_b x$$

The breakdown value of the surface electric field, E_b, is assumed to be 10^4 V/cm.

The plasma parameters for the voltage gradient region may be calculated and are shown in table I. The parameter, h, is included in the equation for T_h to account for the fact that not all of P_s goes into heating of the plasma, and raising the temperature. A heat absorption calculation shows that the heat loss into the dielectric surface constitutes a major sink for the energy in the plasma. The plasma thickness, d, was assumed to be 0.0025 cm, or 1 percent of the length of the voltage gradient region, ℓ. M_a and T_h do not depend on d, but n and ρ_s do. It should also be noted that all four of these parameters are independent of the brushfire velocity, v_b. This is because they all depend on the time-integrated power density, P_s, i.e., the energy, which is independent of velocity. The temperature, T, in the equation for surface resistivity, ρ_S, is a composite of the initial field emission/ low collisional plasma temperature, T_i, and the temperature due to heating, T_h. These two temperature profiles have been combined in the root-sum-square sense:

$$T = (T_i^{\,2} + T_h^{\,2})^{0.5}$$

Since only the T_h component of T depends on h and the T_i component does not, h was selected to give the most reasonable voltage profile, V(x) (see Figure 3a), when computed using equation (15). The value selected was

$$\frac{h}{cg} = 8.71 \cdot 10^{-4},\ h = 1.964 \cdot 10^{-4},\ \text{where } c = 2.71 \cdot 10^{4} \text{ joules/(ev-gram), and}$$

$$g = 8.32 \cdot 10^{-6} \text{ grams/joule}$$

As noted previously, h is a very small fractional number. The term in the expression for f(x) in equation (15):

$$12\left(\frac{cg}{h}\right)^{3/2} Cv_b$$

must be a constant.

This means that the individual parameters may change as long as the value of the above combination remains constant. For example, if the per unit area capacitance C is doubled, the propagation velocity, v_b, is halved. There is no reason to expect c, g, or h to change when C is doubled by halving its thickness. It is possible, however, that c, g, or h may have values different from those assumed here, but their combination, cg/h must remain at the same value.

For all of the computations and parametric curves which will be presented next, the brushfire propagation velocity, v_b, was selected to correspond to that of an ion of mass 16 (oxygen) accelerated through the breakdown voltage, V_b, or a 2-mil sheet of Kapton. The bulk breakdown electric field is assumed to be 10^6 V/cm:

$$v_b = \sqrt{2eV_b/m} = 2.45 \cdot 10^{7} \text{ cm/sec for } V_b = 5000 \text{ V}$$

Figure 3a shows the assumed voltage profile, V(x), which is moving to the left at a velocity, v_b, equal to $2.45 \cdot 10^7$ cm/sec. V drops linearly from 2500 V at x = 0 to zero at $x = \ell$ where ℓ was chosen to be 0.25 cm in order to give the surface breakdown electric field of 10^4 V/cm. Figure 3a also shows the current density, J_S, which increases linearly from zero at x = 0 to 3.18

A/cm at $x = \ell$. Figure 3b shows the power density, P_s, which increases linearly from zero at $x = 0$ to $3.18 \cdot 10^4$ W/cm^2 at $x = \ell$. The plasma ion and electron density, n_i, is also shown in Figure 3b. It varies parabolically from zero at $x = 0$ to $2.03 \cdot 10^{15}$ particles/cm^3 at $x = \ell$. The ionization is assumed to be 10 percent of the total and therefore the neutral particle density is $1.83 \cdot 10^{16}$ particles/cc at $x = \ell$.

Figure 4a shows the temperature, T, and surface resistivity, ρ_s, as a function of x/ℓ.

Figure 4b shows the originally assumed linearly falling voltage profile and the voltage profile computed by using the ρ_s integral in equation (15). It can be noted that V(o) is only 90 percent of V_m at $x = 0$. However, the voltage gradient is greater than the surface breakdown electric field of 10^4 V/cm when x/ℓ is greater than about 0.5. The temperature in Figure 4a is extremely "hot" for small x/ℓ values but cools down quickly as the plasma density increases. A minimum is reached at x/ℓ equal to about 0.4 where the heating effect takes over, and the temperature rises slowly as x/ℓ increases beyond this point. The surface resistivity profile in Figure 4a varies as the inverse three-halves power of T.

In order for the computed voltage to be identical to the assumed voltage profile, the surface resistivity would have to be an inverse function of x:

$$\rho_s = \frac{1}{Cv_b x}, \quad Cv_b \int_x^{\ell} \rho_s dx = \ln \frac{\ell}{x}, \text{ where } e^{-f(x)} = \frac{x}{\ell}$$

The physics of the problem requires initially a very hot plasma and therefore a very small resistivity, rather than the initially very large surface resistivity required by the assumed linear voltage profile. What this says is that the linear voltage profile was not a good assumption. The computed profile of Figure 4b is presumably a better approximation to the "real" propagating brushfire voltage profile. In principle, iteration of the computations performed here with the computed voltage should provide a better solution. This is not done here, and a more thorough analysis using a computer is recommended.

BLOWOUT AND FLASHOVER CURRENTS, G'

The ratio of blowout to flashover currents, G', is a very important parameter in defining the EMI margin of immunity of a spacecraft to arc discharges. The current density, J_s, of 3.18 A/cm calculated in the previous section is that which flows to the point of arc discharge initiation in a plasma sheet and thence directly to the conductive substrate below. This is what has been termed the flashover current. Because of the localized nature of this component, the electric and magnetic fields effects are also expected to be localized. Previously, the only long range effect considered was that due to the displacement current, CdV/dt, where C is effectively the capacitance to space of the arcing element and dV/dt is the time rate of change of the surface voltage. Because C is very small (~pf/cm^2) the corresponding currents are very small, and the voltages induced into cable harnesses were very small and at nonhazardous levels. Blowout currents are additional to the displacement currents discussed above. If they are of appreciable magnitude, they could be a serious source of hazard to spacecraft electrical subsystems.

In this section the results of the previous section on brushfire propagation are used to estimate the blowout current. Both magnetic and electrostatic forces were examined, and the conclusion was reached that only the latter is of consequence. Electric fields normal to the dielectric surface will force electrons to move away in the z direction. The overwhelming majority of electric field lines emanating from the electrons collected from environmental charging land on positive charges induced on the substrate. A few field lines, however, must go off to space to account for the voltage fall-off (or rise) from the dielectric surface potential to the space plasma potential (zero). Thus, it is already clear that the dielectric surface potential, through its associated electric field, plays an important role in determining the blowout to flashover arc discharge current ratio, G'. The magnitude of the electric field for a conducting sphere is

$$E_{radial} = \frac{Q}{4 \pi \epsilon_o a^2} = \frac{V_s}{a} \text{ (MKS units)}$$

where a is the radius of the sphere and V_s is the surface potential and Q is the charge. For an arcing dielectric surface on a real spacecraft, a is not an easily defined parameter and requires a time-dependent NASCAP type of 3-dimensional LaPlace's equation solution in an arc whose discharge charge time is measured in nanoseconds.

We know that a is not as large as the spacecraft dimension and not as small as the dielectric thickness. For our purposes here, we assume that it is comparable to the size of a typical spacecraft box (or 20 cm), but keeping in mind that E_{radial} varies inversely as a.

The fact that edge or punch-through breakdown occurs at -5 kV, but -2.5 kV remains after the discharge, has been ignored up to now except to take the 2.5 kV differential as the voltage which "drives" the brushfire.

Thus:

$$V_s = V_o + V_r + V_m \left(1- \frac{x}{\ell}\right)$$

where V_o is the spacecraft ground potential, V_r is the remaining voltage after the discharge (2500 V) and V_m is the maximum brushfire driving potential (2500 V). The proper signs have to be used to account for the fact that we are considering forces which drive electrons off of the surface. Ions are pulled harder against the surface. For the time being V_o will be assumed to be zero.

The velocity and displacement in the off-surface z-direction for an electron released at z = 0 and t = 0 are given by

$$F_z = eE_z = e\,V_s/a = m\,\frac{dv_z}{dt}$$

Incorporating, as before, the space-time equivalence via the brushfire propagation velocity v_b:

$$v_z(x) = \int_0^x \frac{2eV_m}{mav_b}\left(1-\frac{x}{2\ell}\right)dx = \frac{2eV_m}{mav_b}\,x\left(1-\frac{x}{4\ell}\right)$$

$$z(x) = \frac{2eV_m}{mav_b^2}\int_0^x \left(x-\frac{x^2}{4\ell}\right)dx = \frac{eV_m}{mav_b^2}\,x^2\left(1-\frac{x}{6\ell}\right)$$

The above equations apply in the MKS system of units. If a, v_b, and x are in cgs units, v_z and z may be obtained in cgs units by multiplying both of the above equations by 10^4.

Figure 5 shows v_z and z plotted as functions of x/ℓ. At $x = \ell$, v_z is $3.37 \cdot 10^9$ cm/sec and z is 19.1 cm. These values for electrostatic deflection are about eight orders of magnitude greater than the comparable values caused by magnetic forces on the plasma current.

To calculate the off-surface surface current density, J_{sz}, an integration over x has to be performed:

$$J_{sz}(x_1) = \int_0^{x_1} e\,n(x)\,v_z(x_1-x)\,dx$$

$$\text{where } v_z(x_1-x) = \frac{2eV_m}{mav_b}(x_1-x)\left(1-\frac{x_1-x}{4\ell}\right)\cdot 10^4 \text{ cm/sec}$$

$$n(x) = Ax^2 \text{ electrons/cm}^3 \text{ (x in cm)}$$

$$A = 0.1 \cdot 3.76\cdot 10^{21}\; gCE_b^{\,2}/2d = 3.25\cdot 10^{16}$$

$J_{sz}(x_1)$ is plotted in Figure 8 for $0<x<0.05\ell$.

$$J_{sz}(x_1) = 3.04\cdot 10^4 \left(\frac{x_1}{\ell}\right)^4\left(1-\frac{2x_1}{5\ell}\right) \text{ amp/cm}$$

At $x_1 = \ell = 0.25$ cm, J_{sz} would be 18,240 A/cm, which is much too large in view of the 3.18 A/cm value for J_s (in the x-direction) in the plasma sheet at $x = \ell$. There is, however, a mechanism whereby J_{sz} is cut off at a much smaller value. The situation is that at the same time as the off-surface charge is being evaluated by electrostatic forces, the charge finds itself above a plasma whose Debye length is shorter than its height above the surface of the dielectric. At some height, $\bar{z}$, and Debye length, λ, the electric field due to the charges below becomes completely blocked off, and the effective electric field becomes zero. We assume that this height, $\bar{z}$, is equal to 4.6λ; i.e., when the electric field is shielded by 99 percent.

The effective height $\bar{z}$ (x) is calculated by averaging the z-distance travelled by all of the particles released from x = 0 to $x = x_1$.

$$\bar{z}\,(x_1) = \frac{1}{\int_0^{x_1} n(x)\,dx} \int_0^{x_1} n(x)\, z\,(x_1 - x)\,dx$$

$$\text{where } z\,(x_1 - x) = \frac{2eV_m}{mav_b^2}\,(x_1 - x)^2\,\left(1 - \frac{x_1 - x}{6\ell}\right) \cdot 10^4 \text{ cm}$$

$$\bar{z}\,(x_1) = \frac{eV_m\ell^2}{mav_b^2}\left(\frac{x_1}{\ell}\right)^2\left(1 - \frac{x_1}{12\ell}\right)\cdot 10^3 = 2.29\left(\frac{x_1}{\ell}\right)^2\left(1 - \frac{x_1}{12\ell}\right) \text{ cm}$$

The Debye length is given by

$$\lambda = 6.9\,(T/n)^{0.5} \text{ cm}$$

where T is the temperature in °K and n is in electrons/cm^3. Figure 6 shows $\bar{z}$ and λ plotted for $0 < x < \ell$ (where $\ell = 0.25$ cm). It can be seen that z is much greater than λ for most of the range of x/ℓ except near x = 0. At $x = \ell$, z is about 2 cm, which is about 10 percent of the value for $\bar{z}$, the height of a single electron released at x = 0. Since the temperature for small values of x is nearly completely dominated by the initial high-field-emitted electrons which are cooling off:

$$T \doteq T_i = \frac{1.381\cdot 10^{13}}{n + n_0}\text{ ev} \doteq \frac{1.60\cdot 10^{17}}{n}\ {}^{o}K$$

$$\text{and} \qquad \lambda = \frac{6.9.4.00\cdot 10^8}{n} = \frac{2.76\cdot 10^9}{3.25\cdot 10^{16}x^2} = \frac{8.49\cdot 10^{-8}}{x^2} = \frac{1.36\cdot 10^{-6}}{(x/\ell)^2}\text{ cm}$$

Equating z to 4.6λ:

$$(x_1/\ell)^4 = 2.73\cdot10^{-6},\ x_1/\ell = 0.0407,\ x_1 = 0.0102 \text{ cm}$$

Putting this value for x_1 into the equation for J_{sz} (x_1):

$$J_{sz}\ (x_1) = 3.04\cdot10^4 \cdot 2.73\cdot10^{-6} = 0.083 \text{ A/cm}$$

The blowout to flashover current ratio, G', taken to be the ratio of J_{sz} (x_1) to the maximum value of the plasma sheet current, J_s, (at $x = \ell$) is then $G' = J_{sz}(x_1)/J_s(\ell) = 0.083/3.18 = 0.026$ or 2.6 percent. Figure 7 shows z and 4.6λ plotted versus x/ℓ and their intersection at $x/\ell = 0.041$.

A more nearly correct calculation for J_{sz} involves inserting the Debye shielding effect into the expression for v_z. We consider the shielding to apply to the external electric field by multiplying the potential by the exponential factor so that the corrected off-surface velocity, v_z* is given by:

$$v_z{}^*\ (x_1) = \int_x^{x_1} \frac{2\ e\ V_m\ (1-\frac{x}{\ell})}{mav_b}\ e^{-\bar{z}/\lambda}\ dx$$

Since the x values of consequence are very small ($x/\ell < 0.05$), the above expression may be simplified to

$$v_z{}^* \doteq \frac{2eV_m}{mav_b} \int_x^{x_1} e^{-\bar{z}/\lambda}\ dx$$

From the previous analysis,

$$\bar{z}/\lambda = 2.29\ (x/\ell)^2/(1.36\cdot10^{-6}x^2/\ell^2) = 1.68\cdot10^6\ (x/\ell)^4$$

Figure 8 shows v_z* computed numerically and plotted as a function of x/ℓ. It starts at about 10^8 cm/sec at x = 0 and drops to nearly zero by the time that $x/\ell = 0.04$. The expression for J_{sz} now is

$$J_{sz}\ (x_1) = \int_0^{x_1} en\ (x)v_z{}^*\ (x)\ dx = \frac{2e^2V_mA}{mav_b} \int_0^{x_1} x^2\ dx \int_x^{x_1} e^{-1.68\cdot10^6(x/\ell)^4}\ dx$$

independent of the upper limit of the integral, x_1, for values of x/ℓ greater than about 0.04. This value is

$$J_{sz} = 0.0126 \ \text{A/cm}$$

and the ratio of blowout to flashover currents, G', is

$$G' = J_{sz}/J_z = 0.0126/3.18 = 0.40\%$$

Comparing Figures 7 and 8, it is clear that cutting off J_{sz} at $z = 4.6\lambda$ gives too large a value of x/ℓ and hence too large a value for J_{sz} and G'. From Figure 8, the "correct" values of the parameters for Figure 7 should have been:

$$x_1/\ell = 0.0254, \ \lambda = 2.11 \cdot 10^{-3} \ \text{cm}, \ z = 1.47 \cdot 10^{-3} \ \text{cm}$$

$$z/\lambda = 1.43, \text{ and } e^{-z/\lambda} = 0.24$$

The Debye shielding effect has reduced J_{sz} from an excessively large value, 18,240 A/cm, to a value of 0.0216 A/cm. This latter value leads to a G' of 0.40 percent, which is much smaller than those that have been previously reported by us as well as by others. Another "correction" that should be applied is the fact that Debye shielding does cut off the electrons that are leaving the plasma sheet due to electric fields. However, the potential of the plasma remains unchanged, and thus the electric fields beyond the plasma remain unchanged. Therefore the "escaped" electrons continue to be accelerated by the surface potential even through their number is fixed. Since cutoff occurs at a very small x value ($x/\ell = 0.0254$, $\ell = 0.25$ cm), the accelerating potential is very nearly:

$$V_m + V_r = 2500 + 2500 = 5000 \text{ volts}$$

where V_m is the maximum voltage change, and V_r is the remaining voltage after the discharge.

The surface current density, J_{sz}, by the time the escaped electrons have traversed the whole arcing source then is given by:

$$J_{sz} = Nev_z \text{ where } v_z = \left[\frac{2e(V_m + V_r)}{m}\right]^{0.5} = 4.19 \cdot 10^9 \ \text{cm/sec}$$

N is the number of released electrons per cm^2 and is obtained from $n(x)$ by integration from $x = 0$ to $x = x_1$ or $x/\ell = 0.0254$:

$$n(x) = 3.25 \cdot 10^{16} x^2 \text{ electrons/cm}^3$$

$$N = \int_0^{x_1} n(x)dx = 2.774 \cdot 10^9 \text{ electrons/cm}^2$$

Therefore

$$J_{sz} = Nev_z = 1.86 \text{ A/cm, and } G' = J_{sz}/J_x = 1.86/3.18 = 58.5\%$$

Since the electrons, in increasinq their kinetic energy by 5 keV, have been accelerated in the x-direction as well as the z-direction, the use of the full 5 keV in calculating J_{SZ} is not valid. A particle pushing trajectory calculation for the electrons in the presence of existing electric fields is required. Figure 9 is the author's conception of how the equipotential and electric field lines should appear. The escaping electrons do accelerate through the full 5 keV but the current, properly, should not be termed J_{SZ}. From the "guessed" field configuration it appears that the blowout currents should be travelling at about a 45 degree angle to the surface in the direction of the ignition point.

EFFECT OF SPACECRAFT POTENTIAL ON G'

The importance of external electric fields in determining the blowout to flashover current ratio, G', has been discussed in the previous section. In the analysis, the change in the surface electric field due to the arc discharge was taken into account by the space and time dependence of the surface potential, V_S. However, the reference voltage, the spacecraft potential, V_O, was assumed to be constant at zero volts. In orbit, the blowout of the arc discharge electrons must be compensated by the recollection of an equal number of electrons if the spacecraft potential is to be unchanged. Any inequality between blowout currents and return currents must be "made up" by displacements currents in the following charge balance equation:

$$C_s \Delta(V_s + V_o) + \int_0^t I_z dt = C_o \Delta V_o + \int_0^t I_r dt$$

In the above equation C_S is the capacitance of the arcing element to the remainder of the spacecraft (or to space), and C_O is the capacitance of the spacecraft to space. I_Z is the blowout current from the arcing element, and I_r is the replacement current to the remainder of the spacecraft. Taking the derivative of the equation gives the current balance equation which must be satisfied during the arc discharge:

$$C_s \frac{d}{dt}(V_s + V_o) + I_z = C_o \cdot \frac{d}{dt} V_o + I_r$$

I_z is the blowout current density, J_{sz}, computed in the preceding section, multiplied by an appropriate width dimension. I_r is the integral of all of the replacement current densities collected over the entire exposed surface of the spacecraft. As I_r is collected, it returns to the arcing element via various structural paths on the spacecraft. Obviously, the structural current density is low at remote portions of the spacecraft, and becomes greater as the current flow paths converge towards the arcing element. For this reason, it is to be expected that the potential victims of EMI closest to the arcing source would be the most susceptible.

The point here is that V_o adjusts itself in a time dependent manner to assure that the current continuity equation is satisfied. Since electrons are leaving, V_o will go more positive. If, as assumed, V_o is initially near zero, V_o will become absolutely positive and attract electrons from the environment surrounding it, and repel ions. How far positive it becomes is a function of the surface area of the whole spacecraft, and the accessibility of replacement electrons. The problem is similar to that of computing the spacecraft charging potentials, but on a much shorter time scale—tens of ns rather than minutes.

The availability of electrons in the ambient plasma may be estimated as follows: Assume that electrons may take as long as 1 μs to reach the spacecraft, a sphere of radius, R, of one meter at a potential, V_o, of 1 kV. The radius, r, from which electrons can arrive at the surface in 1 μs is given by:

$$\frac{dr}{dt} = v(r) = \left[\frac{2e}{m} V(r)\right]^{0.5} = \left[\frac{2e}{m} \cdot \frac{Q}{4\pi\epsilon_o r}\right]^{0.5}$$

$$\frac{2}{3}(r^{1.5} - R^{1.5}) = \frac{2e}{m} \cdot \frac{Q}{4\pi\epsilon_o} \; t = \frac{2e}{m} V_o R \quad ^{0.5} t$$

$$r = \left\{\frac{3}{2}\left[\frac{2e}{m} V_o R\right]^{0.5} t + R^{1.5}\right\}^{2/3} = 9.47 \text{ meters for } t = 1\mu s$$

For t = 100 ns, r is 2.44 meters. Assuming that the electron density is $1/cm^3$, a spherical volume, for 1 μs, contains $3.20 \cdot 10^{10}$ electrons or a charge of $5.12 \cdot 10^{-9}$ coulombs. By comparison, a 20 cm wide arcing source, grounded, would have a current I_z of 19 A, and would emit, in 1 μs, a charge of $1.9 \cdot 10^{-5}$ coulombs. This is more than three orders of magnitude more charge than is available.

Another calculation which indicates that the current available is insufficient to "clamp" V_0 utilizes the Langmuir - Mott Smith equation for the attraction of electrons at a Maxwellian temperature, T, to a conducting sphere of radius R:

$$I = 4\pi R^2 J_o (1 + \frac{V_o}{T}) = 22.5 \cdot 10^{-4} \text{ A}$$

for R = 1, $V_o = T = 1$ kV, and $J_o = 1 \text{ na/cm}^2 = 10^{-5} \text{ A/m}^2$

a "resistance," R_o, may be calculated from $R_o = \frac{V}{I} = 4 \cdot 10^6$ ohms

The solution for the blowout current, I_Z, in the presence of a variable time dependent V_0 may be obtained from the following

$$I_z = J_{sz} w; \; J_{sz} = Nev_z; \; v_z = \sqrt{\frac{2eV_s}{m}}; \; V_s = V_r - V_o$$

$$V_o = I_r R_o; \; V_o = \frac{1}{C_o} \int I_c dt$$

In the above equations, w is the width of the arcing source, N is the number of electrons that have been ejected before the Debye shielding cutoff, V_S is the surface potential, V_r is the remaining voltage after the discharge (2500 V), I_r is the resistive replacement current flowing in R_0, and I_C is the displacement current flowing in the capacitance of the spacecraft to space, C_0. The electrical circuit is shown in Figure 10.

The above equations lead to the following result:

$$\frac{t}{\tau} = \frac{1}{p-q} \ln\left(\frac{x-p}{x-q} \cdot \frac{1-q}{1-p}\right) - \ln\left(\frac{x^2 + Bx - 1}{B}\right)$$

where p and q are roots of $x^2 + Bx - 1 = 0$,

$$\tau = R_o C_o, \; x = I_z / I_{zo}, \; I_{zo} = A/V_r^{0.5} = 1.316 \cdot w(\text{cm}) \text{ A},$$

$$A = Ne(2e/m)^{0.5} \cdot 100 \; w = 0.0236 \; w, \; B = R_o I_{zo} / V_r = w \cdot R_o / 1900.$$

Figures 11 and 12 show $I_Z(t)$ and $V_0(t)$ for w = 10 cm. and various values of R_0. The time constant, $\tau = R_0C_0$, varies from 1 ns to 1 μs on the assumption that the C_0 is 100 pf. For R_0 large, V_0 approaches V_r and I_Z decreases because V_S becomes small. For R_0 small, as in many vacuum tank experiments, V_0 never gets very large, and I_Z remains near I_{Z0}. Figure 13 shows the steady state I_Z and V_0 plotted as a function of R_0.

The preceding discussion about R_0 indicates that it is quite large. For the approximation that $I_r << I_c$, the solutions for I_z and V_0 are:

$$I_z = I_{zo}\,[1-t/(2\tau_0)],\ V_0 = V_r[1-(1-\frac{t}{2\tau_0})^2]$$

I_z decreases linearly to zero in a time $2\tau_0 = 2C_0V_r/I_{zO} = 3.8\cdot10^{-7}/w$ seconds or 38 ns for w = 10 cm. V_0 rises parabolically to V_r in the same time period. For a 10 cm square sample, then the brushfire propagates according to our model in a time, t, of:

$$t = \frac{10\ \text{cm}}{2.45\cdot10^7\ \text{cm/sec}} = 408\ \text{ns}$$

I_z, however, lasts for only 38 ns or about 10 percent of the discharge time with an "average" G' of 29 percent rather than the peak value of 58 percent. Thus, the in-orbit G' is of shorter duration and of lower average magnitude as compared to a laboratory determination with R_0 shorted to ground. A proper laboratory experiment should incorporate a high R_0 but should also include an appropriate C_0.

LIMITING MECHANISMS ON BRUSHFIRE PROPAGATION

The question arises as to whether some processes exist whereby the brushfire propagation might be limited. The paper by Aron and Staskus(9) seems to indicate that propagation continues for samples as large as 5058 cm^2. Their samples (4 mil teflon) were laid on an aluminum plate that was 0.313 cm thick. This seems to indicate that the plasma sheet resistance, the part behind the voltage gradient region, is not a problem.

In some applications, the dielectric sheet with the vacuum deposited aluminum (VDA) is not over a good conducting ground plane. In these cases the surface resistivity of the VDA film becomes important. Typical values are in the order of 1 ohm-per-square, but this may be exceeded by more than a factor of 10 after handling and during the installation process. A 100 cm long sample then will develop more than 1 kV with a 1 A/cm arc discharge surface current density, J_s. If one considers then that arc discharge surface currents are really not 1-dimensional, but rather flow from the whole surface towards a single breakdown point, the surface current density increases greatly and therefore the voltage drop may become comparable to the voltage across the dielectric before breakdown. Although the brushfire propagation as developed depends only on the electric field at breakdown, E_b, rather than the voltage, V_b, a dependence on the latter may develop in a more critical analysis.

Figure 14 shows an example of a set of surface voltage measurements before and after an arc discharge. The discharge clearly did not wipe off the stored charge uniformly. The charge seems to have flowed towards the edge at which breakdown occurred, but was slowed down as the distance from that location increased. This particular sample was mounted on an aluminum substrate. However, the VDA was sandwiched with a Kapton sheet between the VDA and the aluminum substrate. Thus, resistive currents were forced to flow through the VDA rather than through the substrate.

SUMMARY AND CONCLUSIONS FROM THE BRUSHFIRE ARC DISCHARGE MODEL ANALYSIS

Summarizing the analytical development of the arc discharge brushfire propagation model should begin with noting the many deficiencies. The first is that the analysis is 1-dimensional while most arcing configurations are 2-dimensional. Thus, no account is taken of the "sidewards" propagation effect both as it affects the brushfire wavefront steepness requirements, and the greater concentration of plasma sheet currents as they converge towards the arc initiation point. There are many assumptions which may or may not be justified such as the ignoring of thermal conductance, and the assumption that the plasma thruster data, $8.32 \cdot 10^{-6}$ gram per joule of material ablated, was applicable. The assumption of a plasma sheet thickness, 1 percent of the length of the voltage gradient region, was not derived from physical principles, but rather, from an idea of what a "sheet" should be. The gram-molecular-weight of the dielectric material, 16, also was a guess, and the specific heat depends on this number. The plasma properties which would clearly identify the time dependent roles of electrons, ions and neutrals have not been carefully treated. In particular, the inertial/collisional role of ions in determining the brushfire velocity should be included in the basic equations so that the velocity is consistent with the other physical processes involved. The areas of improvements that are needed in the present analysis are summarized below. As stated previously, there are many improvements that can be made in the analytical model as presented here, and it is hoped that this work will provide some insight into how a more nearly correct model should be formulated.

- Many assumptions need to be examined
  - Thermal conductivity, mass ablated, plasma sheet thickness, etc.
- More physical processes need to be included
  - Role of ions in determining brushfire velocity; ablation, ionization and radiation processes
  - "Mechanical" processes of particle acceleration and collisions
- Self-consistent solutions are needed
  - Computerized approach
- Model should be expanded to include the 2-dimensional problem

The analysis has provided a first-cut solution to voltage, current, plasma density, temperature and resistivity profiles associated with the plasma sheet of a propagating brushfire wavefront. The flashover surface current density associated with the discharge rises linearly with distance away from the head of the wavefront as

$$J_{sx} = Cv_b V_m x/\ell$$

At the bottom of the voltage falloff region J_s reaches a maximum value:

$$J_{sx} = Cv_b V_m = 3.18 \text{ A/cm, for } V_m = 2500 \text{ V}$$

which is proportional to the breakdown voltage V_m. The duration of the arc discharge is simply the sample size (linear dimension) divided by the brushfire propagation velocity, v_b. To the extent that the theory is applicable to the 2-dimensional case, the duration should be proportional to the square root of the area. The following combination of parameters for a given dielectric material must be a constant:

$$\left(\frac{cg}{h}\right)^{3/2} Cv_b$$

where c is the specific heat, g is the mass ablated per joule, h is the fraction of the power expended in raising the plasma temperature, C is the dielectric capacitance per unit area and v_b is the brushfire propagation velocity. The above combination of parameters must be a constant for a given dielectric material except that C also depends on the thickness. Thus, increasing the thickness decreases C, and hence v_b should decrease correspondingly.

Another result of the analysis is that magnetic V X B forces are much less effective in producing blowout currents than electric field forces. Debye shielding of electric fields limits the blowout electrons to the very tip of the brushfire wavefront. An analogy for the blowout current would be the smoke puffing out of the smokestack of the locomotive of a train as it moves forward -- not the whole train burns. The blowout electrons are accelerated by the chargeup potentials and the ratio of blowout to flashover currents, G', has been calculated to be

$$G' = 58.5\%$$

This value of G' takes into account the experimentally observed fact that about one-half of the stored charge (1/4 of the stored energy) remains after the discharge. If the fraction of remaining charge were lower, the flashover current would be proportionately larger, but the blowout current would be about the same since the number of electrons remains nearly the same and the total accelerating potential also remains the same. Thus G' would decrease, but only by a factor of about two. From the results of the above analysis, G' is independent of the size of the arcing source. The surface voltage at breakdown affects G' as its square-root.

The dependence of the blowout current, and therefore G', on the spacecraft potential is rather drastic, and depends on the capability of the spacecraft to collect return currents, either from the surrounding plasma or from the blowout current itself. The spacecraft potential rises in order to compensate for the blown off charges and to collect the required number of electrons, or to make up the deficiency via displacement currents. Because the spacecraft capacitance to space, D_o, is small (~100 pf), the accelerating potential for the blowout electrons is quickly cancelled -- in 38 ns out of a total of 408 ns for the whole brushfire process to take place -- in our example of a 10 cm square arcing source. Most laboratory experiments in the past have grounded the arcing source to the vacuum system ground through a low resistance of a few ohms. A more proper simulation of in-orbit conditions for arc discharges would be to increase the grounding resistance to greater than 10,000 ohms, and add a parallel capacitance of about 100 pf. The conclusions resulting from the brushfire model analysis are summarized below:

- The flashover surface current density, J_{sx}, (3.18 A/cm), is proportional to V_m.
- $(h/cg)^{3/2} \cdot Cv_b$ is a constant (see text for definition of parameters).
- The discharge duration is proportional to the length of a 1-dimensional source.
  - And is proportional to the square-root of the area of a 2-dimensional source.
- The blowout surface current density, J_{sz}, (1.86 A/cm), is proportional to the square-root of the surface potential at breakdown.
- G' (58.5 percent) is independent of the area of the arcing source.
  - Depends on electric field forces; magnetic forces are negligible.
- G' is grossly affected by how the spacecraft potential varies during the discharge.
  - J_{sz} is cut off by positive spacecraft potentials (smaller net potentials) during the discharge.
- Laboratory measurements of G' should take into account conditions on orbit.

The author acknowledges the contributions of two colleagues to the present analysis of the arc discharge brushfire propagation model. J. M. Sellen, Jr. coined the term, "brushfire," and formulated the initial concepts on the steepness requirements for a propagating wavefront. R. L. Wax critiqued many aspects of the model. In particular, his insight into the plasma physical processes was invaluable.

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Table 1. Plasma Parameter Resulting from a Linear Voltage Gradient

$J_s = Cv_b V_m x/\ell = Cv_b E_b x$	$\rho_s = \frac{K}{d} T^{3/2} = 12(\frac{cg}{h})^{3/2}[\ell n(1+\frac{x^2}{A})]^{-3/2}$
$P_s = J_s \frac{\partial V}{\partial x} = J_s E_b = Cv_b E_b{}^2 x$	$T_h = -\frac{h}{c_m}\int P_s dt = \frac{h}{cg}(1 + x^2/A)$
$M_a = \int gP_s dt = gCE_b{}^2 x^2/2$	where $A = 2M_o/(gCE_{bb}{}^2) = 1.70 \cdot 10^{-8} cm^2$
$n = 3.76 \cdot 10^{22} gCE_b{}^2 x^2/(2d)$	and T_h is the temperature due to heating.

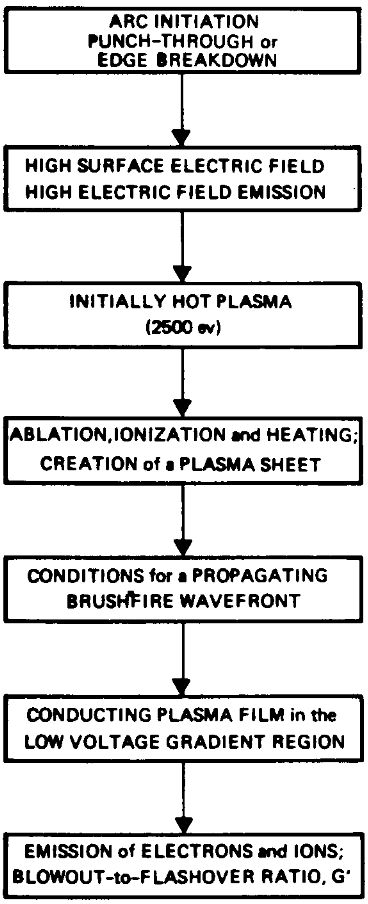


Figure 1. Overview of the Brushfire Propagation Analysis

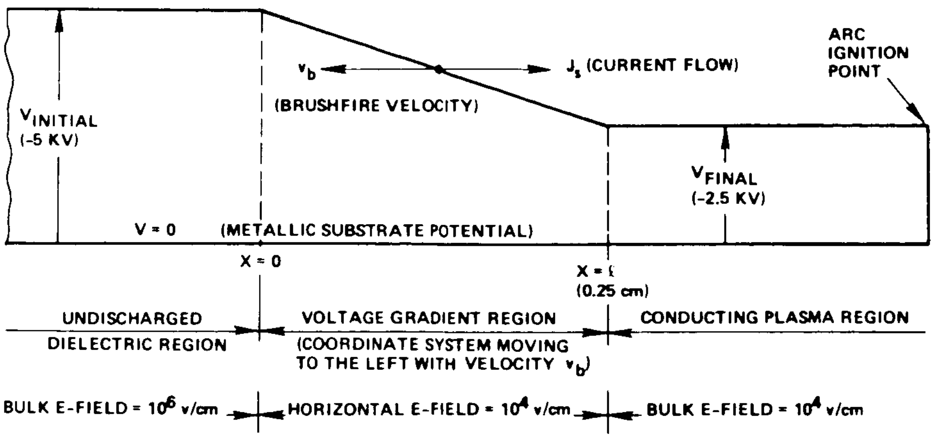


Figure 2. Voltage Profile of a Propagating Brushfire Wave Front

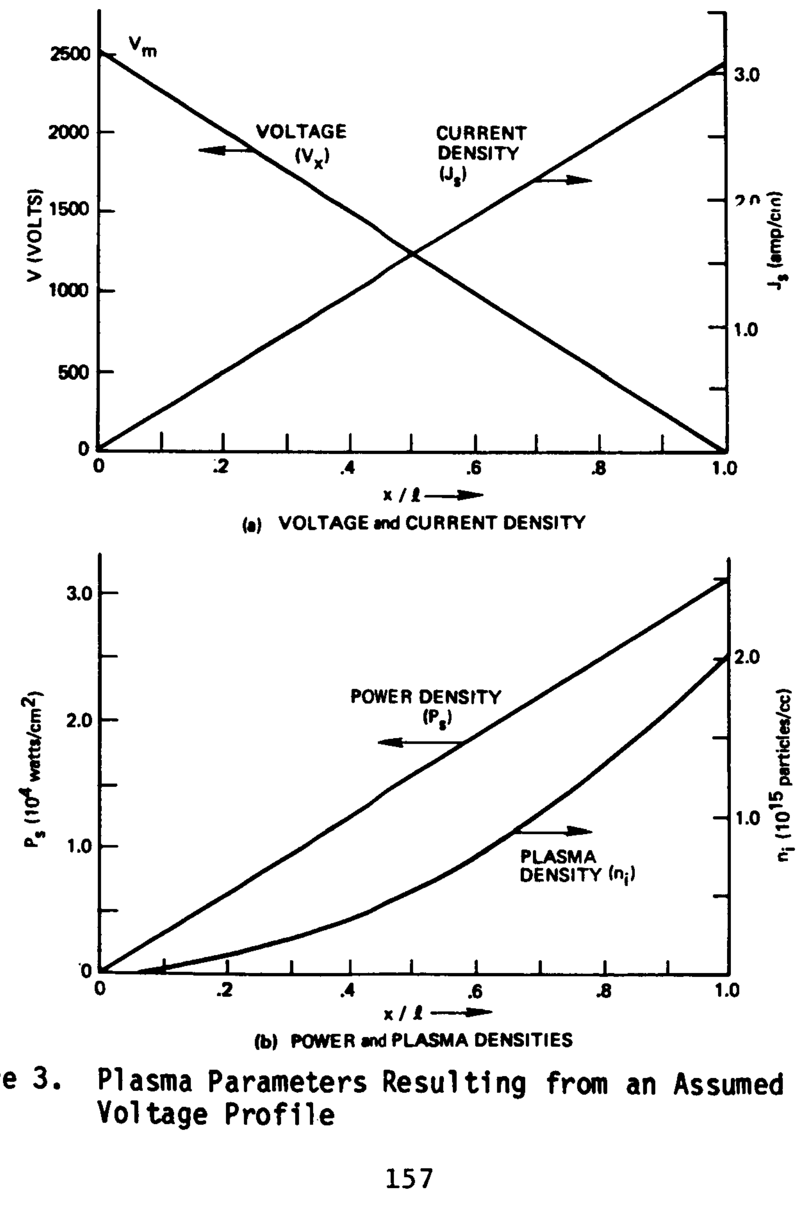
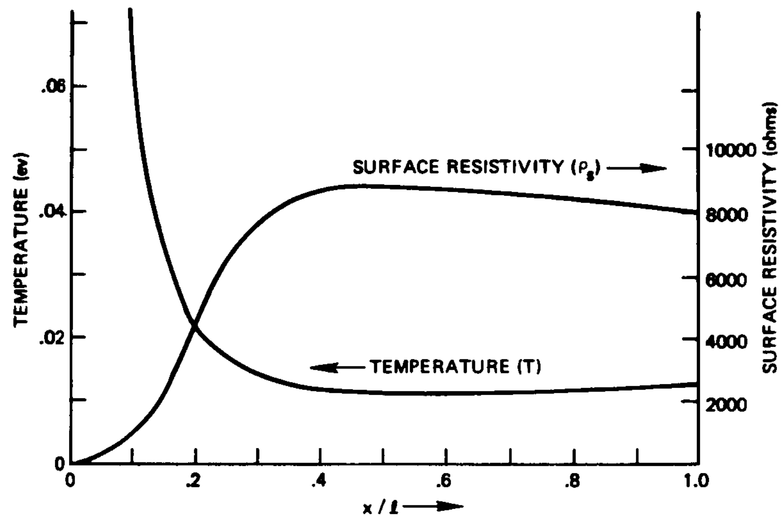
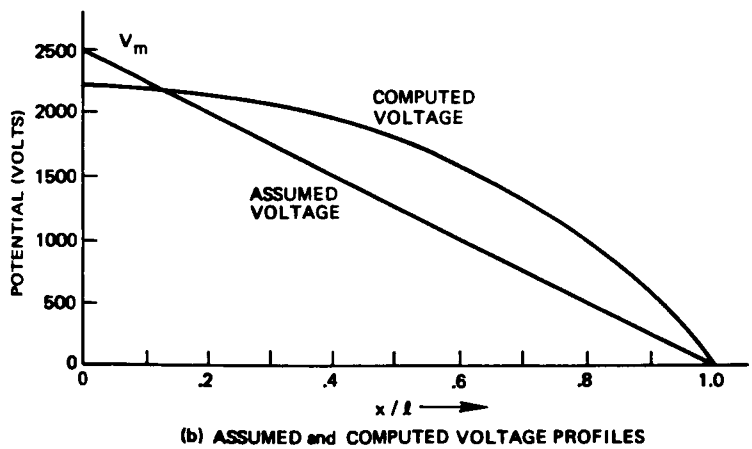


Figure 3. Plasma Parameters Resulting from an Assumed Linear Voltage Profile



(a) TEMPERATURE and SURFACE RESISTIVITY PROFILES



(b) ASSUMED and COMPUTED VOLTAGE PROFILES

Figure 4. Additional Plasma Parameters Resulting from a Assumed Linear Voltage Profile

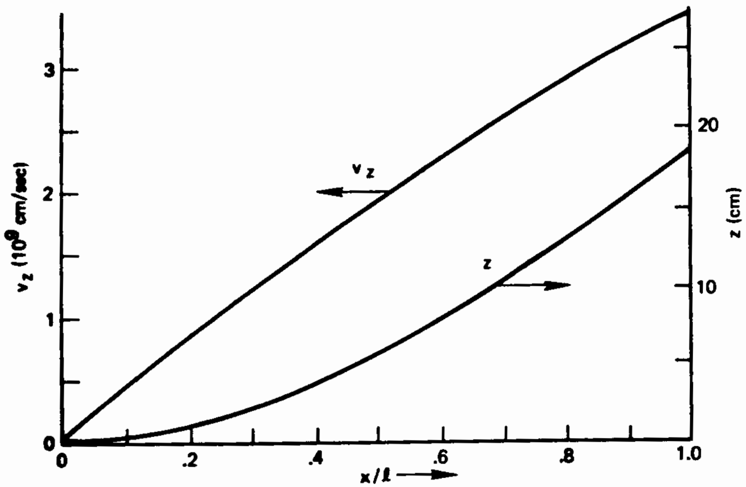


Figure 5. v_z and z for Electron at x = o
(No Plasma Shielding)

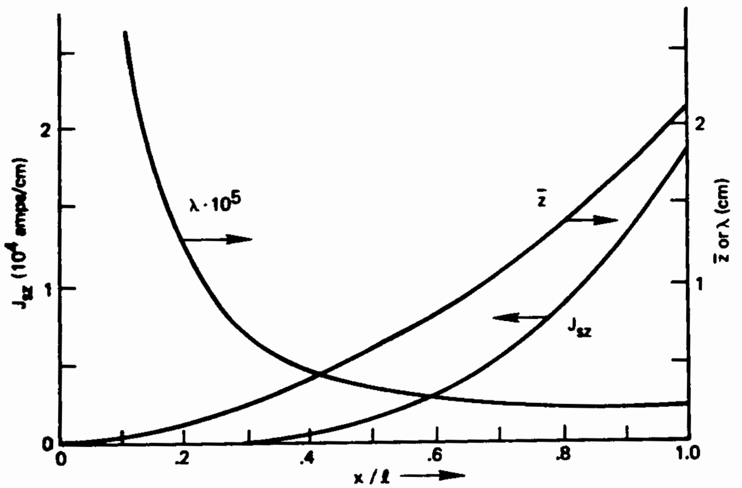


Figure 6. Debye Length (λ), J_{sz} and z
(No Plasma Shielding)

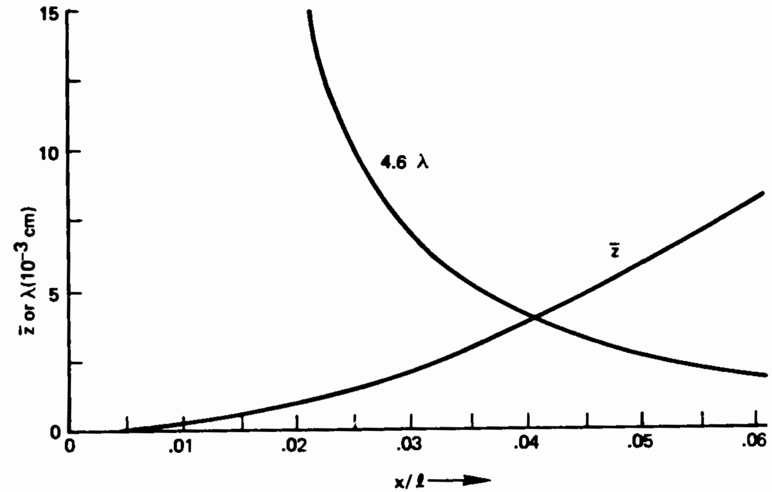


Figure 7. 4.6λ and z versus x/ℓ

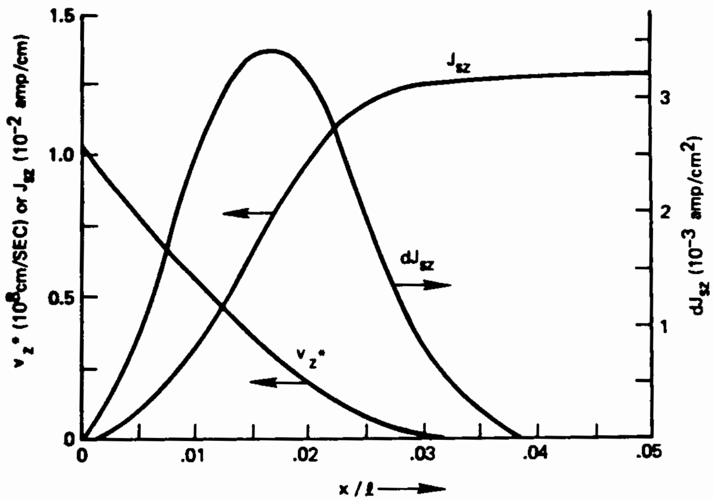


Figure 8. v_z*, dJ_{sz} and J_{sz} Versus x/ℓ
(Shielding by Plasma)

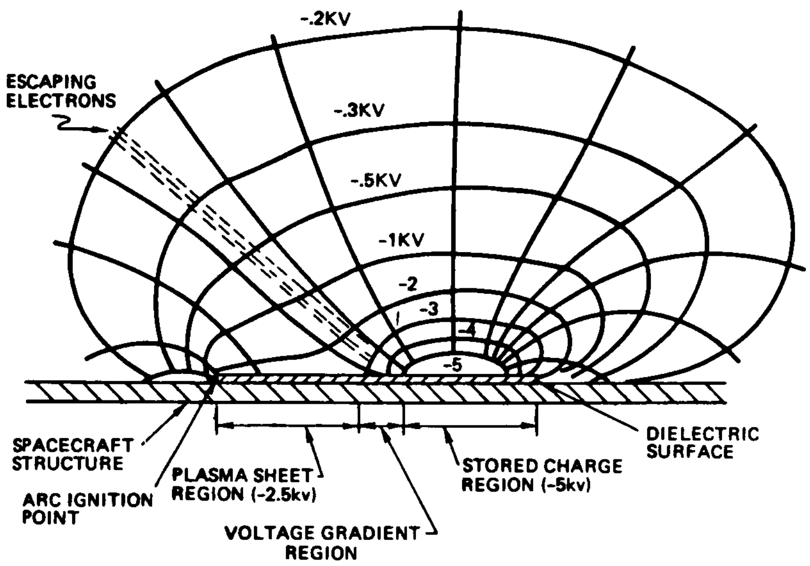


Figure 9. Brushfire Equipotential and E-field Lines

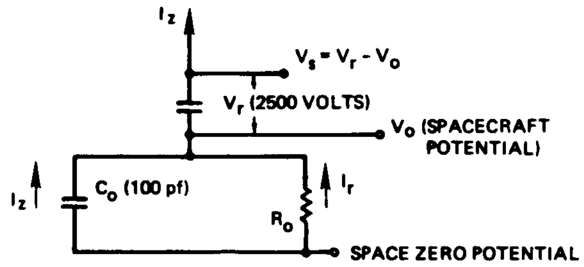


Figure 10. Electrical Circuit Defining I_Z and V_0Ct

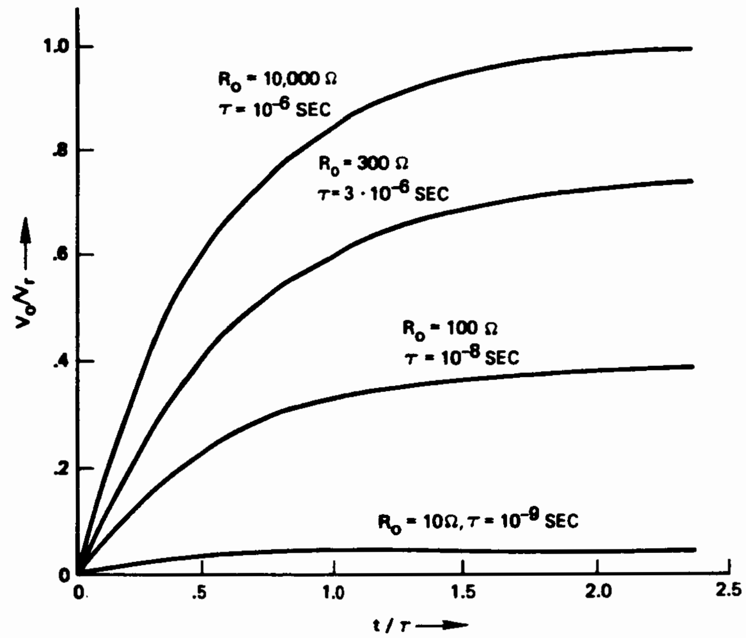


Figure 11. V_o/V_r as a Function of t/τ

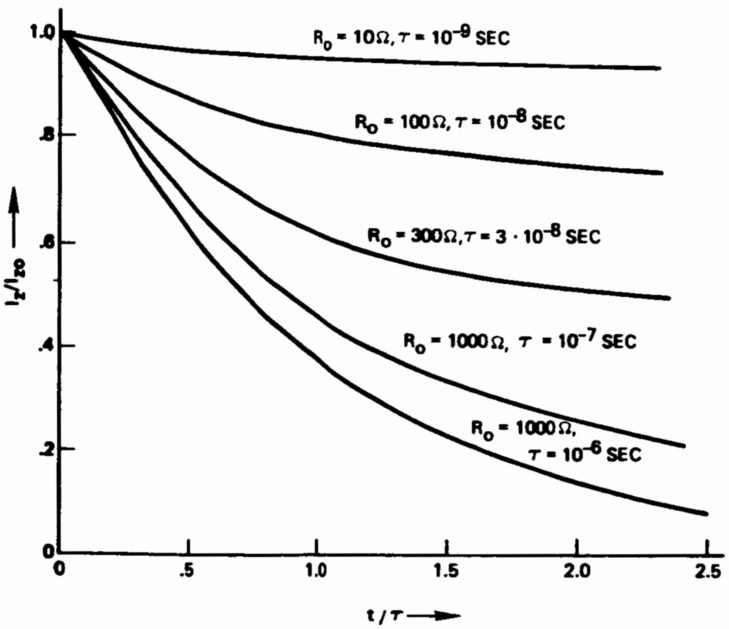


Figure 12. I_z/I_{ZO} as a Function of t/τ

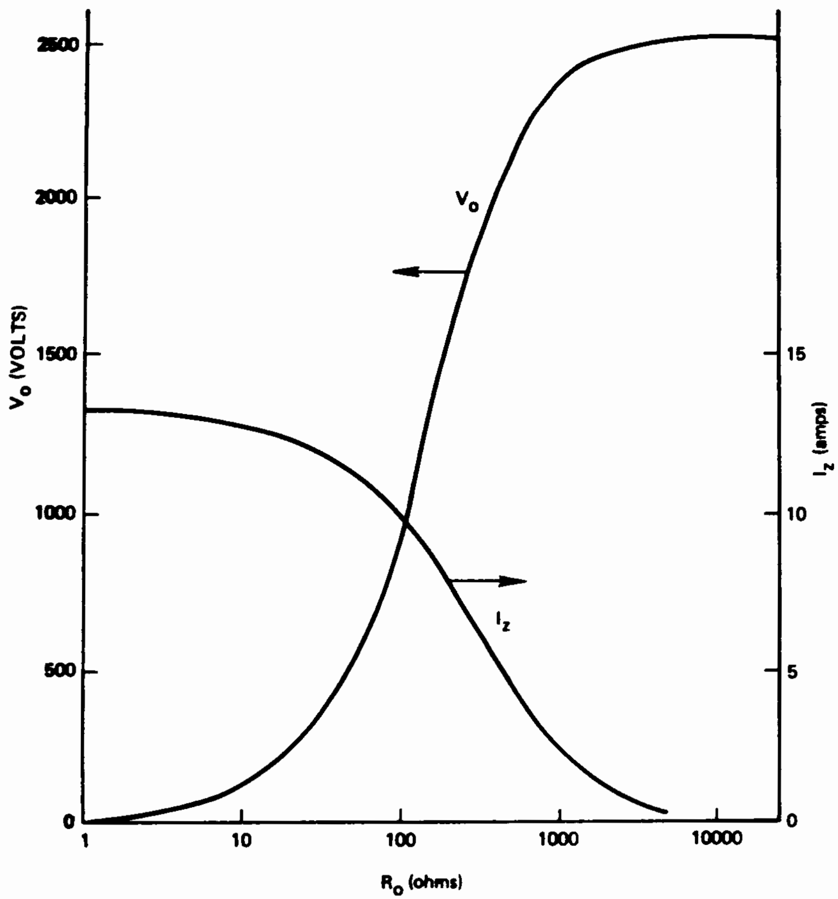


Figure 13. Steady State V_o and I_z Versus R_o for W = 10 cm

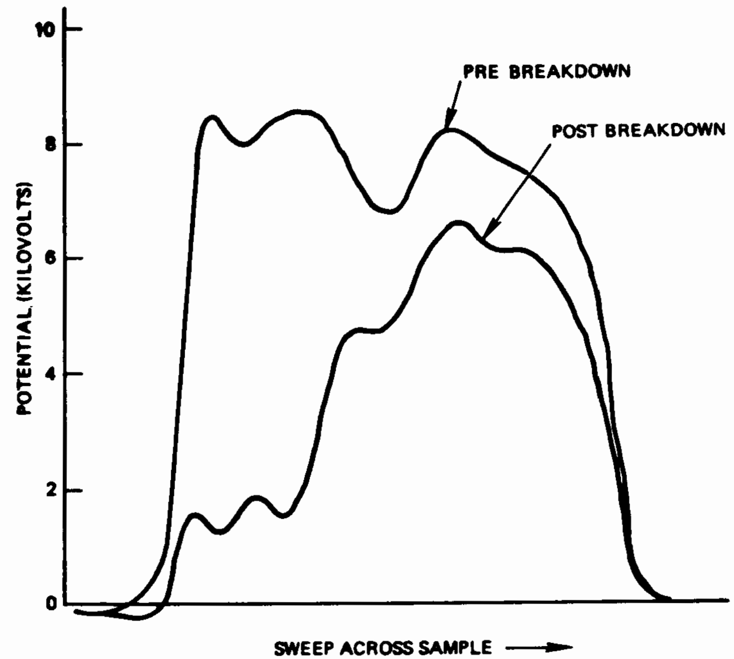


Figure 14. A Potential Profile of 6 x 6 inch Kapton Laminate Sample Before and After a "Relatively" Low Voltage Breakdown Near Edge of Sample