

A SIMPLE MODEL OF ELECTRON BEAM INITIATED DIELECTRIC BREAKDOWN*

B. L. Beers, R. E. Daniell, and T. N. Delmer
Beers Associates, Inc.
Reston, Virginia 22090

We have developed a steady state model which describes the internal charge distribution of a planar dielectric sample exposed to a uniform electron beam. The model includes the effects of charge deposition and ionization of the beam, separate trap-modulated mobilities for electrons and holes, electron-hole recombination, and pair production by drifting thermal electrons. If the incident beam current is greater than a certain critical value (which depends on sample thickness as well as other sample properties), the steady state solution is non-physical. We interpret this to mean that above the critical beam current, the sample breaks down.

INTRODUCTION

This paper describes a simple model of a beam charging experiment. The motivation for the model is the need to understand low voltage breakdown such as that which occurs in dielectric material exposed to the radiation environment of space (ref. 1). Our approach to the problem is motivated by the work of O'Dwyer (ref. 2) on high voltage breakdown.

The model configuration is shown in figure 1, and it is assumed to have reached a steady state. A pair of infinite grounded plates are separated by an infinite, homogeneous dielectric, and a spatially infinite electron beam is incident on the arrangement normally, through one of the plates. The beam causes ionization at one rate, I , and deposits electrons at a second rate, S . The fact that these rates are constant forces the solution to the problem to have symmetry about the centerplane between the plates. All variables are either symmetric or anti-symmetric about this plane. The problem considered here is simple, but sign conventions must be handled carefully to avoid confusion.

We take current to be positive when it is directed toward the right. Consequently, an electron beam traveling to the right represents negative current which we denote by J_B . Since the beam is losing electrons at the rate S (electrons $\text{cm}^{-3}\text{s}^{-1}$), the magnitude of J_B is decreasing but

$$\frac{dJ_B}{dx} = eS > 0 \quad (1)$$

where e is the magnitude of the electronic charge. Because of the build up of negative charge, the electric field (E) is positive (i.e., directed toward the

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right) in the left half of the dielectric ($0 < x < L/2$), vanishes at the midplane, and is negative in the right half of the dielectric. The electrons and holes produced by the beam drift under the influence of the electric field. We denote the resulting conduction current by J_C . The total current is the sum of the beam current and the conduction current. In steady state, conservation of charge requires

$$\frac{d}{dx} (J_B + J_C) = 0 \quad (2)$$

Since J_C has the same symmetry properties as E , the solution of equations (1) and (2) is

$$J_C = (1 - 2x/L) eSL/2 \quad (3)$$

For the geometry as we have defined it, this solution is independent of the ionization rate I , the mobilities of the charged species, and all other parameters except sample thickness and electron deposition rate.

ELECTRON AND HOLE BEHAVIOR

The conduction current is the sum of the electron and hole currents which are defined in the usual manner:

$$j_n = -nev_n = ne\mu_n E \quad (4)$$

$$j_p = pev_p = pe\mu_p E \quad (5)$$

where n and p are the electron and hole densities, v_n and v_p are the electron and hole drift velocities, and μ_n and μ_p are the trap modulated mobilities of electrons and holes. The electron and hole currents have the same symmetry properties as J_C and E . Because of the symmetry of the problem, we will consider only the left half of the dielectric ($0 < x < L/2$). In this case, only J_B , v_n , and ρ (the net charge density) are negative. E , j_n , j_p , n , and p are all positive.

With the use of equations (1) - (5), one variable can be expressed in terms of the others. Solving for p results in

$$p = -\mu_n n / \mu_p + (1 - 2x/L)SL / (2E\mu_p) \geq 0 \quad (6)$$

Transport equations can be written for electrons and holes:

$$\frac{d}{dx}(nv_n) = vn + (I + S) - knp \quad (7)$$

$$\frac{d}{dx}(pv_p) = vn + I - knp \quad (8)$$

where k is the recombination rate and v is the collision ionization coefficient. Because of equation (6) the equation for holes (eq. 8) is redundant. There is no time derivative because of the assumption of steady state, so the gradient of the electron flux is equal to the three source and sink terms on the right. Note that each unsigned term on the right-hand side of equation (7) is positive: the first term is the avalanche (i.e., collision ionization) term, the second is the beam ionization and charge deposition, and the third is recombination.

The only other equation needed is that for the electric field:--

$$dE/dx = e(p - n)e \quad (9)$$

where ϵ is the dielectric constant of the sample material. The symmetry forces $E(x) = j_n(x) = j_p(x) = 0$ at $x = L/2$ which are the boundary conditions.

Using equations (4) - (7), we obtain the following equation for the behavior of the electron current:

$$\begin{aligned} -dj_n/dx &= +vj_n/(\mu_n E) + (I + S)e \\ &+ [j_n/(e\mu_n E)]^2 ek\mu_n/\mu_p \\ &- (1 - 2x/L) kSLj_n/(2\mu_n\mu_p E^2) \end{aligned} \quad (10)$$

None of the variables in this equation are negative. Therefore, in the region under consideration, all terms on the right-hand side have the sign explicit in front of them. The last two terms in combination are negative as in equation (6). The equation governing the electric field is equation (9). Equation (6) can again be used to eliminate p and equation (4) to eliminate n with the result that

$$\begin{aligned} dE/dx &= -[(1 + \mu_p/\mu_n) j_n/(\mu_p E) \\ &- (1 - 2x/L) eSL/(2\mu_p E)]/e \end{aligned} \quad (11)$$

We have adopted the form of the collision ionization term given by reference 3:

$$v = \alpha_0 \mu_n |E| \exp(-E_0/|E|) \quad (12)$$

where α_0 has units of inverse length.

Equations (10) and (11) are a pair of coupled ordinary differential equations for j_n and E . They may be cast into dimensionless form by the use of the dimensionless variables

$$u = 1 - 2x/L \quad (13a)$$

$$\xi = -j_n/eSL \quad (13b)$$

$$\eta = E/aE_0 \quad (13c)$$

and the dimensionless constants

$$a = eSL^2/(4\epsilon\mu_p E_0^2) \quad (14a)$$

$$b = \epsilon k/e\mu_n \quad (14b)$$

$$c = \mu_p/\mu_n \quad (14c)$$

$$\lambda = \alpha_0 L/2 \quad (14d)$$

$$\sigma = I/S \quad (14e)$$

The two equations (10) and (11) become

$$d\xi/du = \lambda \xi \exp(-1/a\eta) |\eta|/\eta + (1 + \sigma) - b\xi(u - \xi)/\eta^2 \quad (15)$$

$$d\eta/du = [(1 + c)\xi - u]/\eta \quad (16)$$

The boundary conditions are $\xi(0) = \eta(0) = 0$.

PROPERTIES OF THE SOLUTIONS

Note that $(u - \xi)/\eta$ is the dimensionless hole density while ξ/η is the dimensionless electron density. The requirement that these two densities be non-negative places a constraint on ξ :

$$0 \leq \xi \leq u, \text{ for } 0 \leq u \leq 1 \quad (17)$$

Any solution which falls outside this range is not physically meaningful.

Let us first consider the case for which there is no collision ionization ($\lambda = 0$). Then the two equations (15) and (16) have the solution

$$\xi(u) = cu \quad (18)$$

$$\eta(u) = gu$$

where c is the solution of

$$(1 + \delta - b) c^2 - [(1 + \delta)(1 + \sigma) + 1 - b] c + (1 + \sigma) = 0 \quad (19)$$

and g is defined by

$$g^2 = [(1 + \delta) c - 1] \quad (20)$$

If the solution is to be physically meaningful, then c must be less than unity. For many situations of interest, both δ and b are very small so that $c = 1 - \Delta$ where Δ is much less than unity. To second order in the small quantities δ and b

$$\Delta \approx \delta(1 - \delta - b/\sigma) \quad (21a)$$

$$g^2 \approx \delta b/\sigma \quad (21b)$$

For purposes of illustration, however, it is preferable to use values of order one. In figure 2 we show the solution for $\delta = 0.5$ and $b = \sigma = 1.0$. The solution is independent of a , the dimensionless electron deposition rate. Both the electric field and the two currents (electron and hole) are linear. The electron and hole densities are constant. The quantity ϕ is the dimensionless potential and is defined as

$$\phi(u) = 2V(x)/(aE_0L) = \int_0^u \eta(u) du - \phi_0 \quad (22)$$

where $V(x)$ is the electrostatic potential with $V(0) = V(L) = 0$, and ϕ_0 is the dimensionless potential at $u = 0$ ($x = L/2$).

In the preceding example the dimensionless electron and hole currents (ξ and $u - \xi$, respectively) as well as the dimensionless electric field (η) are independent of the dimensionless electron deposition rate (a). (Of course, the

dimensional quantities are strongly dependent on the electron deposition rate.) However, when λ is non-zero, the collision ionization term introduces an explicit dependence on a . Because this term is always positive, its presence causes both $d\xi/du$ and $\xi(u)$ to increase. We have solved equations (15) and (16) numerically for the same values of b , b , and σ as above, but with $\lambda = 0.01$. As expected, the solution is no longer independent of a . Solutions for three values of a are shown in figure 3. Note that for $a = 4.58$ both the hole current and the hole density vanish at the electrodes. If a is increased beyond 4.58, the solution predicts negative hole densities near the electrode. Since negative hole densities are physically meaningless, this means that there is no steady state solution for $a > 4.58$. We interpret this to mean that the dielectric will break down.

DISCUSSION

We have presented a simple model of the effects of an electron beam on a dielectric sample. We have assumed that the sample is homogeneous and that the incident beam is spatially uniform. We have also assumed that the beam deposits electrons uniformly throughout the sample. We have found that if the incident beam current (or electron deposition rate) becomes larger than a critical value, there are no steady state solutions, which we interpret to be an indication of breakdown.

We have only begun to explore the properties of this model for realistic values of the model parameters. We anticipate that the simplicity of the model will limit the accuracy with which it represents a real dielectric charging problem. However, we hope that the very simplicity of the model will make it possible to thoroughly study and understand the physical processes leading to breakdown in this idealized case. We feel that this is an important first step in the development of more realistic models which take into account material inhomogeneities (e.g., localized defects).

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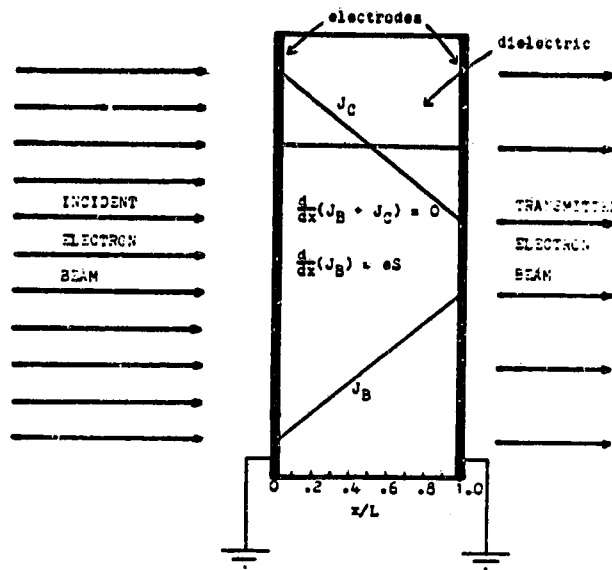


Figure 1. - Model Geometry. Electrons moving to the right produce negative current. The beam deposits electrons uniformly throughout the sample. These electrons drift toward the two electrodes where they return to ground. In steady state the total current flowing to ground is equal to the difference between the transmitted beam current and the incident beam current.

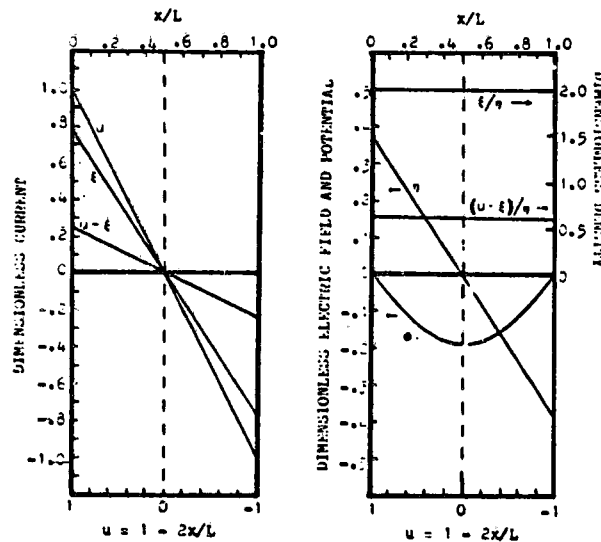


Figure 2. - Solution for no collision ionization ($\lambda = 0$), $\delta = 0.5$, $b = \sigma = 1.0$. Total conduction current is proportional to u . The electron current (ξ) and the hole current ($u - \xi$) are constant fractions of the total conduction current. The electron and hole densities [ξ/n and $(u - \xi)/n$] are also constant. n is the dimensionless electric field, and ϕ is the dimensionless potential.

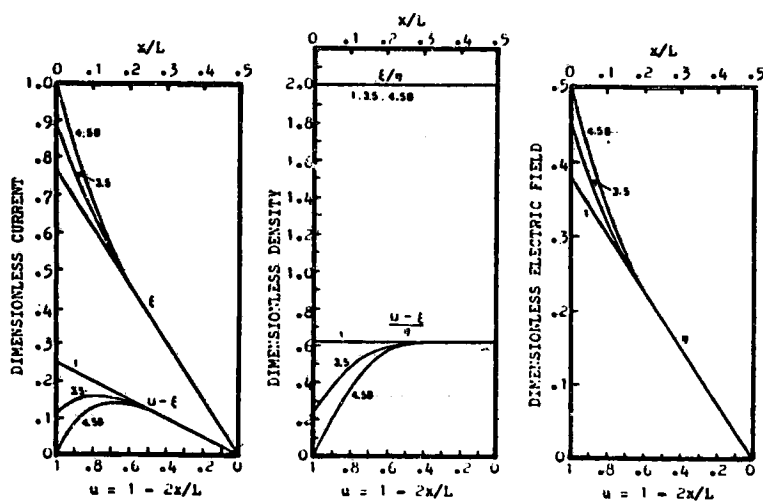


Figure 3. - Solutions for $\lambda = 0.01$, $\delta = 0.5$, $b = \sigma = 1.0$, and $a = 1, 3.5$, and 4.58 . When the dimensionless electron deposition rate $a > 4.58$, the solution gives negative hole densities $[(u - \xi)/\eta]$ near the electrodes. This implies that there is no physically meaningful steady state solution, i.e., the dielectric breaks down.