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ELECTRIC FIELD EFFECTS ON ION CURRENTS IN SATELLITE WAKES*

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Small currents associated with satellite spin, dielectric conduction, or trace concentrations of H^+, can have a substantial effect on the potential of a satellite and the particle currents reaching its surface. The importance of such small currents at altitudes below about 300 km stems from the extremely small O^+ currents impinging on the wake-side of the spacecraft.

The focus of the present study is the particle current on the downstream side of the AE-C satellite. Theoretical estimates based on a newly described constant of the motion of a particle indicate that accounting for small concentrations of H^+ remove a major discrepancy between calculated and measured currents.

1. INTRODUCTION

Many studies, both theoretical and experimental, have been made of the interaction between a satellite and the near earth plasma (refs. 1-11). The present study concerns charged particle current on the wake-side surface of a spacecraft in the earth's ionosphere, where the vehicle is mesothermal; its speed V_0 exceeds the thermal velocity of plasma ions but is much less than the thermal velocity of the ambient electrons.

In our analysis calculated currents are compared with those that have been observed by Samir et al. (ref. 10) on the Atmosphere Explorer C (AE-C). The AE-C experiments are well suited to our purpose, since its rate of spin, as well as the plasma densities, constituents and temperatures were known. Moreover, measurements were conducted at night, thus avoiding complications associated with active solar arrays.

Measured satellite voltages were in the range $V \sim 9\text{-}10\Theta$, where the electron temperature Θ is in electron volts. These results exceeded theoretical estimates based on balance between ion and electron current on a conducting surface by a factor ~2.0-2.5. Theoretical surface potentials were also substantially less than expected from ion-O^+ current balance at each point of a dielectric surface. In a previous work it was shown that accounting for either rotating currents of charge embedded in the dielectric or small concentrations of H^+ suffice to bring theory and experiment into conformance (ref. 12). The calculated voltages reported in Reference 12 will be used in the calculations of current given in Section 3.

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The reported charged particle densities in the ambient plasma encountered by AE-C are in the range 10^4-10^6 cm^{-3}, with equal electron and ion temperatures $\Theta \sim 0.1$ eV, corresponding to ambient Debye lengths $\lambda_D \leq 3$ cm. The ionic components of the plasma are the singly charged species of atomic oxygen, O^+, and atomic hydrogen H^+. Our primary focus will be in the altitude regime below about 300 km where O^+ is the dominant ion in the ambient ionosphere and in the far wake trailing the satellite. In the highly evacuated wake region near the surface of the satellite, however, H^+ may be the dominant ion (ref. 13).

Particle densities and currents, especially in the rarefied wake downstream of the satellite, are the most difficult to determine. Indeed the question of wake structure is the most intensively studied aspect of interactions between a spacecraft and the ionosphere (refs. 4-11). In the quiescent environment of the equatorial ionosphere, where satellites can develop potentials of several times Θ, ion currents to the satellite surfaces facing upstream are little affected by electric and magnetic fields and may be calculated in the neutral approximation (ref. 1), as if particles moved in straight lines with constant speed. The manner of estimating current to surface elements on the wake-side of a high Mach number vehicle is much less clear. One might suppose, for example, that electrical forces may attract substantially greater currents than estimated on the basis of straight line orbits. In Section 2, we will invoke constants of the motion in an axi-symmetric potential field to determine that O^+ number and current densities at the wake-side pole of a non-emitting sphere at an altitude of a few hundred kilometers are several orders of magnitude above neutral approximation densities. The wake-side O^+ currents remain small, however, relative to H^+ currents, and calculations presented below show that electric fields suffice to increase the H^+ currents by orders of magnitude to the observed level.

Two of the dynamical constants used in the calculation are the energy and the axial component of angular momentum; the third is a less well known constant of the motion which applies for potentials of the form $V_0(r) = f(\theta)/r^2$ where θ is the polar angle and r the distance in a spherical co-ordinate system. It reduces to the total angular momentum in the limit of spherically symmetric potentials.

The general physical assumptions underlying the model developed in the subsequent sections of this paper are that

1. The plasma is collisionless and quiescent.

2. The geomagnetic field has a negligible effect on particle motion in the spacecraft sheath.

3. In the plasma rest frame, the unperturbed ions have Maxwellian velocity distributions with finite, equal temperatures.

4. Ions are neutralized on impact with a surface.

5. The spatial dependence of its electron density distribution is related to the space potential V, through the Boltzmann factor $\exp(eV/\Theta)$.

Discussion of further simplifying approximations of a more special character occurs at the point in the text where they are introduced.

2. ION CURRENTS

The normal component of ion current density at a surface element located at $\vec{r}_s$ on the body is given by

$$j = -e \int_{v.n<0} \vec{v}.\vec{n}\ f(\vec{r}_s, \vec{v})\ d^3\vec{v} \tag{1}$$

where $\vec{n}$ is the outward normal at $\vec{r}_s$. The distribution function $f(\vec{r}, v)$ at the phase space point $\vec{r}$, $\vec{v}$, satisfies, in general, the Vlasov-Poisson system of equations. For a perfectly absorbing body f satisfies the boundary condition $f(\vec{r}_s, \vec{v}) = 0$ for $\vec{v}.\vec{n} > 0$.

Calculations of current to a satellite surface often use the assumption that ion currents to the satellite are given by the neutral approximation, which neglects the influence of electric fields. This assumption is quite good at the front (upstream) surface where ions reaching the satellite have energies (~5 eV for O^+) substantially larger than electrical potential energies. The situation is less clear on the wake-side where electric fields may substantially enhance particle and current densities over the neutral approximation values. In the following paragraphs, this problem is addressed by formulating bounds on $j(\vec{r}_s)$ and $n(\vec{r}_s)$, and applying these bounds for an assumed, non-self-consistent model potential.

The normal current density at a point r_s on the surface where the potential is $V(\vec{r}_s)$ can be written ($m_i = e = 1$)

$$j(\vec{r}_o) = \int \mu_o\ f(\vec{r}_s, \vec{V}_o)\ v_o^3\ dv_o\ d\ \Omega_o$$

$$= \int \mu_o\ f_o(\vec{v}) \left(\frac{1}{2} v^2 - V(\vec{r}_s)\right) dv^2 \left|\frac{d\vec{\Omega}_o}{d\vec{\Omega}}\right| d\vec{\Omega} \tag{2}$$

Here $\vec{v}_o = (v_o, \vec{\Omega}_o)$ is the velocity of a particle at $\vec{r}_s$, $\mu_o = -\vec{\Omega}_o \cdot \vec{n}(\vec{r}_s)$ $(1 \geq \mu_o \geq 0)$, and $\vec{v} = (v, \vec{\Omega})$ the velocity at $r = \infty$ on the trajectory that connects to the phase space point $(\vec{v}_o, \vec{r}_s)$. If

$$|d\Omega_o / d\Omega| < 1 \tag{3}$$

then

$$j \leq j_b = 2\int f_o(\vec{v}) \left(\frac{1}{2} v^2 - V(\vec{r}_s)\right) v\ dv\ d\ \vec{\Omega}\ , \tag{4}$$

that is, j_b is an upper bound on the normal ion current density at $\vec{r}_s$. Similarly the particle density satisfies

$$n < n_b = \sqrt{2} \int f_o(\vec{v}) \left|\frac{1}{2} v^2 - V(r_s)\right|^{1/2} v\ dv\ d\ \vec{\Omega} \tag{5}$$

The bounds established here require the inequality (3); that is, in terms of inside-out trajectories, neighboring orbits emanating from $\vec{r}_0$ with a given energy must diverge more at $r = \infty$ than at their point of origin. Although the inequality (3) appears to be a reasonable assumption for attractive potentials, the general conditions under which it applies have not been established.

For the following considerations, we take a spherical satellite in the potential $V(r,\theta)$ where r and θ are spherical polar coordinates with polar axis in the direction of $\vec{V}_0$. We consider model potentials of the form

$$V = -r^{-2} f(\theta) + V_0(r) \tag{6}$$

where $V_0(r)$ is a spherically symmetric potential. The asymptotic, far wake potential has in fact this form with (ref. 1)

$$V_0(r) = 0$$

$$f(\theta) \sim M^2 a^2 \cos^{-2}\theta \exp(-M^2 \tan^2\theta) \tag{7}$$

where a is the radius of the satellite. For the discussion below the particular form of $f(\theta)$ is arbitrary, however, and may be chosen to fit potentials near the satellite.

The utility of the potential of the form in equation (6) is that a particle moving in it possesses three constants of motion. In addition to energy and the component of angular momentum about the polar axis the quantity

$$C = \frac{L^2}{2} - f(\theta) \quad , \tag{8}$$

where L is the magnitude of the angular momentum about $r = 0$, is conserved along a particle trajectory. This follows readily upon taking the scalar product of $\vec{L} = \vec{r} \times \vec{V}$ with both sides of the torque equation ($m_i = e = 1$)

$$\frac{d\vec{L}}{dt} = -\vec{r} \times \nabla V \tag{9}$$

taking account of equation (6) for V. There are fewer constants of motion than for $V = V(r)$, since in the latter case the direction of L as well as its magnitude is constant. The dynamical constant C of equation (8) is a rigorous constant for potentials of the form (6), and should not be confused with the invariants used by Samir and Jew (ref. 14) and criticized by Laframboise and Whipple (ref. 15).

The effective potential for radial motion of a particle is now given by

$$V_{eff}(r) = V_0(r) + \frac{C}{r^2} \tag{10}$$

With this V_{eff} we can solve for the orbits and evaluate the bounds on ion current and particle densities at the satellite surface. The orbit equations are particularly tractable for the interesting case of particles which reach the poles of the sphere, especially the wake-side pole. For these particles the axial component of angular momentum is zero, the orbits are planar, and the solution of the equations of motion is reducible to quadratures.

From

$$E = \frac{1}{2} v^2 = \frac{1}{2} \dot{r}^2 + \frac{C}{r^2} + V_0(r) = \frac{1}{2} v_0^2 + V_0(a) - \frac{f(\theta_0)}{a^2} \tag{11}$$

$$C = \frac{1}{2} a^2 v_0^2 (1 - \mu_0^2) - f(\theta_0) = \frac{1}{2} r^4 \dot{\theta}^2 - f(\theta) \tag{12}$$

we obtain

$$\int_{\theta_0}^{\theta} \frac{d\theta'}{[C + f(\theta')]^{1/2}} = \pm \int_a^{\infty} \frac{dr}{r[Er^2 - C - r^2 V_0(r)]^{1/2}} \tag{13}$$

for the final direction θ of a particle launched from $r = a$, $\theta = \pi$ in the direction μ_0 with speed v_0. Here μ_0 is the cosine of the angle between $\vec{v}_0$ and $\vec{r}_0$.

Consider now a positive energy particle with initial coordinate $\theta_0 = \pi$. For attractive potentials, the orbits will appear as indicated in figure 1.

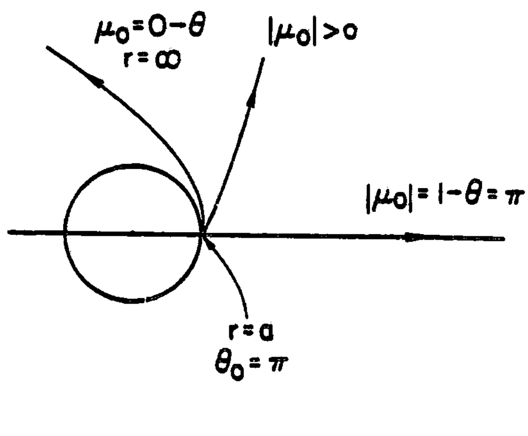

Figure 1. Schematic diagram of inside-out orbits starting on the sphere $r = a$ at $\theta_0 = \pi$.

The range of θ integration for calculating the bounds of equations (4) and (5) is generated by variations of μ_0^2 between 0 and 1. Here we have assumed that the particles are not attracted back to the surface. This affect however can only decrease the particle flux at the point of interest, and therefore does not influence the nature of equations (4) and (5) as upper bounds.

To proceed further, we now specialize to potentials of the form

$$V_0(r) = 0$$

$$\frac{f(\theta)}{a^2} = \frac{4}{3\pi^2} |V(\pi)| \left(\theta^2 - \frac{\pi^2}{4}\right), \qquad \frac{\pi}{2} \le \theta \le \pi \tag{14}$$

$$= 0 \qquad\qquad 0 \le \theta \le \frac{\pi}{2}$$

which approximates the surface potentials given by Parks and Katz (ref. 12). The final angle at $r = \infty$ on the inside-out trajectory corresponding to $\mu_0 = 0$ is given by

$$\theta_\ell = \frac{\pi}{2} \left|\frac{3E}{V(\pi)}\right|^{1/2} \ln \frac{\left(1 + \left|\frac{V(\pi)}{E}\right|\right)^{1/2} + 2\left|\frac{V(\pi)}{3E}\right|^{1/2}}{1 + \left|\frac{V(\pi)}{3E}\right|^{1/2}} \tag{15}$$

and the limiting angle for the outside-in orbit is

$$\alpha = \pi - \theta_\ell \tag{16}$$

When the velocity distribution remote from the spacecraft is Maxwellian, j_b and n_b are given by

$$j_b = \frac{\sqrt{2}}{\pi^{1/2}} \frac{N_0}{V_0 v_T} \int_0^\infty e^{-\frac{V_0^2 + v^2}{2v_T^2}} \left(\frac{1}{2} v^2 + |V(\pi)|\right) g(v)\, dv \tag{17}$$

$$n_b = \frac{1}{\pi^{1/2}} \frac{N_0}{V_0 v_T} \int_0^\infty e^{-\frac{V_0^2 + v^2}{2v_T^2}} \left(\frac{1}{2} v^2 + |V(\pi)|\right)^{1/2} g(v)dv \tag{18}$$

where

$$g(v) = \exp[vV_0 \cos\alpha(v)/v_T^2] - \exp[-vV_0/v_T^2] \tag{19}$$

3. RESULTS

Numerical integration for the case $M = V_0/v_T = 8$, $|V(\pi)| = 15\ v_T^2$ gives for O^+ ions

$$\frac{j_b}{N_0 e v_T} = 1.55 \times 10^{-10}$$

$$\frac{n_b}{N_0} = 2.64 \times 10^{-11}$$

The last ratio is to be compared with the density ratio

$$\frac{n}{N_0} = 6.22 \times 10^{-16}$$

for the case of zero electric field. Thus, although the effect of electric field may be to yield densities several orders of magnitude larger than those obtained in the neutral approximation, they remain extremely small in comparison with ambient ion densities, amounting to about 1 m^{-3} for $N_0 \sim 10^{11}\ m^{-3}$. By way of further contrast, the ion density ratio is also small compared with the electron density ratio

$$\frac{n_e}{N_0} = e^{-15} = 3.06 \times 10^{-7} .$$

Let us now calculate the effect of electric fields on the density of hydrogen ion current striking the wake side of the satellite at $\theta = \pi$. The result of this calculation is intended to throw some light upon the issue raised by Samir and Fontheim (ref. 16) in their comparison between measured current ratios $I(\phi)/I(90^\circ)$ and those calculated from Parker's model (refs. 17,18). Parker's model is based on solutions of the Poisson equation in which ion densities are determined by particle tracking techniques. Only one species of ion is treated, however, and its mass is the mean mass of ions in the plasma. Samir and Fontheim contend that the two-to-three-order of magnitude discrepancy between measured and calculated current ratios might be removed (1) by properly treating the separate ionic components of the plasma, or (2) by considering the non-steady nature of the plasma environment. The results of our calculations, summarized in Table 1 below, indicate that proper treatment of the hydrogen component of the plasma suffices to remove the discrepancy between theory and experiment.

Several observations are in order. First, Table 1 shows that both the measured ratio r and the estimated upper bound on the ratio exceed the ratio estimated from the neutral approximation, which one may reasonably expect to be a lower bound on r. Second, the measurement at 160° should give a somewhat greater ratio than would be observed at 180°; the neutral approximation

Table 1. Comparison of Measured and Calculated Current Ratios[a]

Case[b,c]	2	3	5	6
$N(H^+)/N(O^+)$[d]	1.28(-3)	1.07(-3)	1.40(-3)	4(-2)
$[j(180)/j(j(90)]_{N.A.}$[e]	8.8(-6)	7.4(-6)	2.8(-6)	2.8(-4)
$jb(180)/j(90)$[f]	1.5(-2)	1.3(-2)	1.7(-2)	0.48
$[j(160)/j(90)]_{EXPT.}$[g]	1.49(-2)	2(-2)	5.8(-3)	4(-2)

(a) Numbers in parentheses give power of 10 by which adjacent entries are multiplied.
(b) Case identification is the same as that in Table 2 of Reference 15.
(c) Case 1 is not considered because no value for hydrogen density was reported in Ref. 10. Case 4 is not considered because of the order of the magnitude difference between ion densities reported in Tables 2 and 3 of Reference 10.
(d) Hydrogen ion to oxygen ion ratios are taken from the Bims measurements given in Table 3 of Reference 10.
(e) The ratio of H^+ and O^+ currents at 180° and 90°, respectively, calculated in the neutral approximation.
(f) The ratio of H^+ and O^+ currents at 180° and 90°, respectively, where jb is the bound on H^+ current calculated from equation (17) using a $V_O/V_T(H) = 2.83$.
(g) The ratio of measured currents at 160° and 90°.

determines that the former would be only about 20 percent greater than the latter. Third, the measured current is well below the estimated upper bound in cases 5 and 6, slightly below it in case 2, and slightly above it in case 3.

That the measurements yield a value slightly in excess of the estimated upper bound could be attributed to the approximate nature of the potential used in the calculations, to the fact that the current probe was at a distance of about 0.5 R_O from the surface, to uncertainties in the in situ H^+ density, or possibly to other factors. We believe nevertheless that the results in Table 1 are a strong indication that accounting for H^+, while ignoring the non-steady character of the plasma, suffices to remove the major discrepancies between measured and calculated currents incident on the wake side of the AE-C satellite.

4. SUMMARY

To determine the effect of electric fields on the wake-side ion currents, we have developed an expression for the upper bound on the current density normal to an element of surface. To be a rigorous upper bound it is required that the Jacobian $|d\Omega_0/d\Omega|$ be less than unity. Utilizing the bounding expressions it is shown for a non-self-consistent model potential that the particle and current densities of O^+ ions at $\theta = \pi$, though substantially enhanced by electric fields over neutral approximation values, still constitute an effect that is small in comparison with the effect of spin for the case of AE-C. Finally, accounting for the effects of electric fields on the small concentrations of H^+ in the ambient plasma appear sufficient to remove the major discrepancies between measured and calculated currents on the wake side of the satellite.

The effect of a magnetic field in the absence of electric fields can only be to reduce ion currents incident on a surface, and therefore cannot account for the vehicle ground potentials observed on AE-C (ref. 12). The combined effect of electron and magnetic fields is not considered in this paper.

For different ionospheric satellites in different environments, for example, in polar environments, the relative importance of the various physical effects may differ from that found for AE-C in the conditions we investigated. Thus for satellites subjected to fluxes of energetic auroral electrons, field enhancement of wake-side collection could be a substantial effect.

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