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4. An Altitude-Dependent Spacecraft Charging Model

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Abstract

A model for the altitude dependence of the hot plasma parameters responsible for the electrostatic charging of spacecraft has been developed. Based upon plasma plasma-orbit theory, the directed velocity is a function of the ambient magnetic field flux density. A consequence of this approach is that while the thermal velocity distributions (assumed to be Maxwellian) of the plasma particles are independent of the magnetic field strength (and hence altitude), the particle densities increase with magnetic field strength. Thus, according to this Model, while the equilibrium voltage is independent of altitude, the charging current density increases With decreasing altitude. However, the probability of such spacecraft charging decreases with decreasing altitude.

1. INTRODUCTION

Almost all of the published data and analyses of spacecraft charging[1,2] have been concerned with spacecraft in geosynchronous orbit ($r \approx 6.6\ R_E$). It is theoretically expected that the charging phenomenon can occur at other altitudes as well. For spacecraft in earth orbit at other altitudes, the characteristics of such charging becomes of practical concern. The purpose of this study Was to develop an analytical model Which yielded the significant parameters of the spacecraft charging phenomenon as functions of altitude above the earth.

2. ALTITUDE DEPENDENCE

The major environment which has a strong altitude dependelice at geosynchronous orbit is the earth's magnetic field. If this field is taken as approximately that of a dipole, its magnetic flux density may be written

$$B = \frac{B_o \sqrt{1 + 3 \sin^2 \lambda_M}}{\left(\frac{r}{R_E}\right)^3} \tag{1}$$

where.

B_o = the magnetic flux density at the surface of the earth at the maghetic equator ≈ 0.3 gauss = 3×10^{-5} webers/m^2;

λ_M = the magnetic latitude (degrees);

r = distance from the magnetic center of the earth;

R_E = radius of the earth.

It is necessary for r and R_E to be in the same units, and that $r \gtrsim R_E$.

It is well known that a plasma can exist in a magnetic field only if the plasma energy density exceeds the magnetic field energy density. The energy density (E. D.) of the quasistatic magnetic field is $B^2/2\mu$, where μ is the permeability of the medium Which contains the magnetic field. For free space $\mu = \mu_o = 4\pi \times 10^{-7}$ H/m (MKS units). Thus far the dipole-approximated geomagnetic field,

$$E.D._B = \frac{B_o^2 (1 + 3 \sin^2 \lambda_M)}{2 \mu_o \left(\frac{r}{R_E}\right)^6}$$

$$= 3.58 \times 10^{-4} \left[\frac{1 + 3 \sin^2 \lambda_M}{\left(\frac{r}{R_E}\right)^6}\right] \frac{\text{joules}}{m^3} . \tag{2}$$

Any piasma whlch is able to extst in such a magnetic Pteld must possess an energy density at least this large.

The energy density of a two-component electrically neutral Fully ionized non-relativistic plasma may be written

$$E.D._p = KE \text{ (directed)} + KE \text{ (thermal)} \tag{3}$$

where

$$\frac{\text{(directed)}}{N} = 1/2\,(m_+ + m_-)\,v\,(dir)^2\ ;$$

$$\frac{\text{KE (thernial)}}{N} = 1/2\, m_+ V_+ (th)^2 + 1/2\, m_-\, v_-\, (th)^2 .$$

In these equations, m_+ and m_- are the rest masses of the positively charged arid negatively charged plasma particles, respectively, while v(dir) and v(th) are the directed and the thermal velocities of those particles... N is the spatial density of each type of particle. As long as the plasma moves as an entity, the + and - particles Will have the same directed velocities. In addition, if the plasma is in thermal equilibrium, the + and - particles will have the same average thermal energied, leading to the relatibnship

$$\frac{v_-(th)}{v_+(th)} \approx \sqrt{\frac{m_+}{m_-}}\ . \tag{4}$$

This last relationship is only an approximation, as measurements in the hot plasmas responsible for spacecraft charging seem to shbw that the temperatures of the + component (largely protons) are about twice the temperatures of the - component (electrons).

The characteristics of the hot (spacecraft charging) plasmas at geosynchronous orbit show that the directed velocity geherally lies between the thermal velocities of the eleetrons and the protons. Thus, the directed velocity is supersonic for the protbns but subsonic for the electrons. Since m, ≈ 1836 m_-, the energy density for such plasma is essentially all due to the directed motion of the protons. To illustrate this, for a plasma with a 5 keV temperature $v_+(th) \approx 108$ cm/sec while $v_-(th) \approx 4.3 \times 109$ cm/sec. Taking v(dir) as 6 X 16e cm/sec (an intermediate Value) yields directed energies of $E_+ \approx 180$ keV and $E_- \approx 0.1$ keV. Only if v(dir) is not greater than $v_+(th)$ is the plasma energy density not largely due to the directed motion of the heavier (+) component.

With $E.D._p \approx 1/2\, m_+\, v(dir)^2$, it *is* possible to estimate how low in altitude a hot plasma moving radially toward the earth cain get before its energy density Is not sufficient to prevent the geomagnetic field from tearing it apart, Equating $E.D._B$ and $E.D._p$ yields

$$r \approx R_E \left[\frac{B_o}{v(dir)}\right]^{1/3} \left[\frac{1 + 3\sin^2 \lambda_M}{\mu_o m_+ N}\right]^{1/6} . \qquad (5)$$

The results of this calculation are shown graphically in Figure 1 for $\lambda_M = 0^o$ (magnetic equator). These results are ad Bpproximatibn because of many factbrs, but due to the steep radial gradient of the energy density of the geomagnetic field,

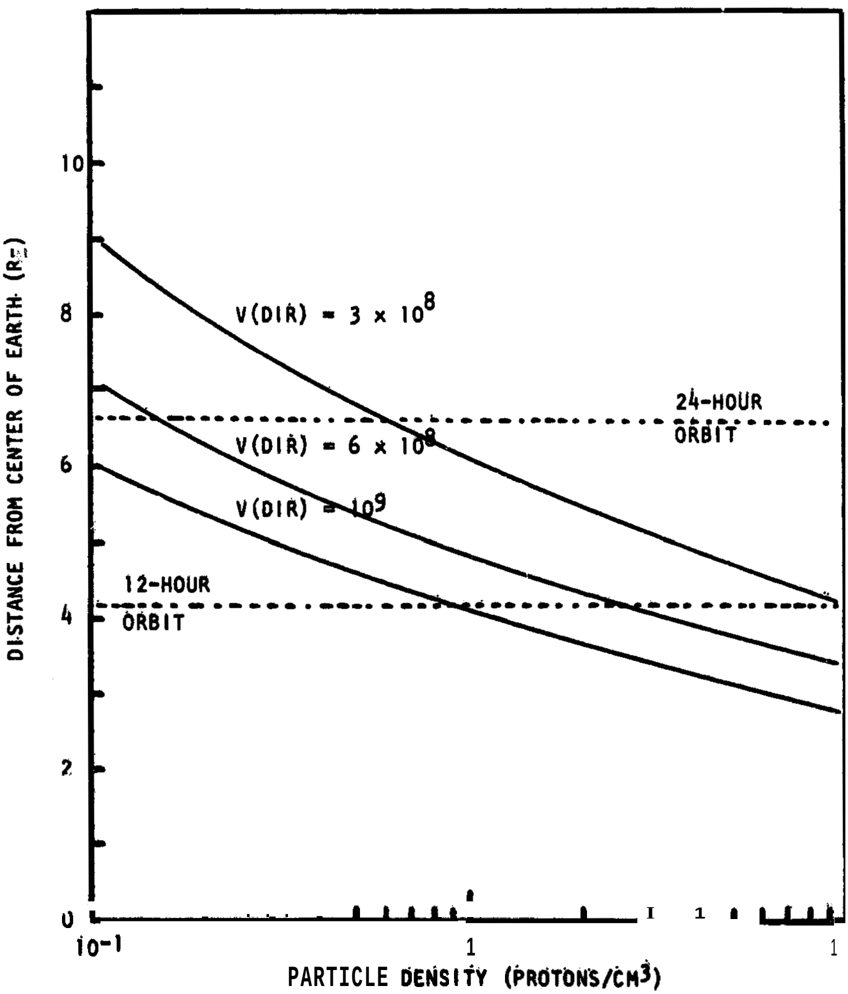

Figure 1. Calculated Hot Plasma Altitude Limit

the approximations produce ohly relatively small perturbations in the results. It is seen that a moderately dense ($\gtrsim$ 16 particles/cm^3) plasma with a reasonably large directed velocity ($\gtrsim 3 \times 10^8$ cm/sec) can reach the 12 hr circular orbit ($r \approx 4.15\ R_E$), while an increase in either partible density ot directed velocity can result in spacecraft charging at even lower altitude earth orbits.

3. MODELING

There are various approximate ways of modeltng a plasma to make plasma problems mathematically tractable. Each model deals with that portion of plasma properties which are most pertinent to the problem at hand while suppressing or ignoring less relevant properties. Plasma orbit theory is among those approximate! models often used in situations in which the motions of individual plasma particles are important. (Hydrodynamic theory is often used for situations in which large-scale plasma oscillations are more important than individual particle motion.) The basis of orbit theory is the conservation of the angular momentum of individual particles about an axis (for example, the direction of the magnetic flux lines). Motion of the plasma particles parallel to the lines of magnetic: flux is not affected (to a first approximation). Thus, it is possible to separate the thermal and the directed motions of the plasma particles by Considering them to be parallel and perpendicular to the magnetic field, respectively. This simplified Model results in a decrease in the directed velocity as the distance from the earth decreases (for example, as the magnetic field strength increases) while leaving the thermal velocity unchanged. It is obvious that this model neglects the compressional heating of the plasma, which certainly is a major factor in the production of the hot plasma in the first place. However, attempts tb incorporate compressional heating resulted in a model in which the plasma energy density increases as rapidly as the geomagnetic field energy density, with the Consequence that a plasma able to exist at any altitude could theoretically reach any other (lower) altitude. This is clearly at variance With observational data (spacecraft charging anomalies have not been reported for low--altitude earth orbits).

Based upon the simplified application of plasma orbit theory, it is possible to write

$$v(dir) = \bar{v}_{th} \cos \alpha \tag{6}$$

where α is the pitch angle (angle between the velocity vector and the direction of the geomagnetic field). This is similar to the equation of motion for Van Allen belt particles. The second invariant for such particles ieads to the relationship

$$\sin^2 a = \frac{E}{B_{max}} \quad . \tag{7}$$

By combining these last two equations, one can obtain

$$v_{dir} = \bar{v}_{th} \sqrt{1 - \frac{\dot{B}}{\bar{B}_{max}}} \quad . \tag{8}$$

According to this mbdel the average particle thermal Velocity (plasma temperature) dbes not change with altitude while the directed velbctty gradually decreases, reaching zero when $B = B_{max}$. At that point, the plasma energy density has been reduced to the energy density bf the geomagnetic field.

The negative voltage to which a bbdy immersed in a plasma of a given temperature will change is **−3.76** times the average electron kinetic energy (E) in electron volts. The negative voltage is due to the fact that the electrons have much higher thermal velocities than the protons, **and** therefore impact the spacecraft surface **much** more often.

The factbr of **3.76** *is* due **to** the fact that the negative current to an unillumi-nated spacecraft Surface varies exponentially with spacecraft voltage (to a first approximation) while the positive current varies linearly with voltage. **This** is a consequence of the fact that the electron motion is subsohic while the proton motion is supersonic. Mathematically,

$$J_+ = J_{op} \left(1 + \frac{V}{V_{op}}\right) \tag{9}$$

$$J_- = J_{oe}\, e^{-V/V_{oe}} \tag{10}$$

where J_+ and J_- are the surface current densities as a function of spacecraft voltage (V), and J_{oe} and J_{op} are those current densities (including the effects of secondaries) when $V = 0$. V_{op} and V_{oe} are the average prbton and electron kinetic energies in electron volts, respectively. Since $m_+ \approx 1836\, m_-$, $J_{oe} \approx \sqrt{1836}\, J_{op} \approx 42.8\, J_{op}$. The value of V will increase until $J_+ = J_-$, assumtng no electrical discharges take place, Equating J_+ to J_- and solving for V yields

$$V = V_o \left[3.76 + \ln\left(\frac{V_{op}}{V_{op} + V}\right)\right] \le 3.76\, V_{oe} \quad . \tag{11}$$

Since the secondary current compbnents vary wlth spacecraft voltage differently than the primary currents do, this factor of 3.76 can be appreciably different in many situations. However, since the particles responsible for the charging have an approximately Maxwellian energy distribution, it lo understandable that a plasma wtth an average electron temperature of V_{oe} (volts) would charge a spacbcraft to voltages $V > V_{oe}$,

4. CHARGING PARAMETERS

Since, according to this model, the particle thermal velocities are unchanged by the directed motion of the plasma toward the earth, the equilibrium Voltage which a spacecraft surface will reach in the plasma will not depend upon altitude. In the absence of electrical discharges, a spacecraft in a 12 hr circular orbit ($r \approx 4.15\,R_E$) will 'herefore theoretitally charge tb the same potential as one in a 24 hr geosynchronous orbit ($r \approx 6.6\,R_E$). However, the! surface current densities (which determine hbw fast the spacecraft will charge) will be a function of altitude. The initial primary surface current density as a function of altitude may be estimated by calculating the limiting plasma particle dedsity of the plasma just as its directed motion has ceased. Since the average particle energy (and the particle energy distribution) remains unchanged according to this model, the particle density must increase linearly with the energy density bf the geomagnetic field.

The calculated numbers derived based upon this model are shown in Table 1. At each altitude r the average energy density of the geomagnetic field (dipole approximation) was taken as the average thermal energy dendity of a stationary plasma at that altitude. Assuming half of this thermal energy density was due to the electrons, the product $\overline{E}_- \cdot N_-$ was bbtained. For each value of limitihg spacecraft potential V, the average electrbn energy was obtained by dividing by 3.76. The electron density (cm^{-3}) was the quotient of $\overline{E} \cdot N_-$ divided by $E_{\dot{e}}$ ($E_- \equiv E_e$ here). The initial primary electron rrent density (J_-) was obtained from the equation

$$J_- = N_- \cdot q \cdot \overline{v} \quad . \tag{12}$$

The results listed in Table 1 are also shown graphically in Figure 2. It is seen that the calaulated initial current density increases rapidly as orbit altitude decreases. This altitude dependence decreases the importance of the phbtoelectric current density (which has an altitude-independent value $< 1\,nA/cm^2$) at lower altitudes. Another consequence of this altitude dependence is that if the spacecraft surface cannot withstand the equilibrium potential to which the plasma can charge it, the rate at which electrical breakdowns (discharges) occur will be much greater at lower altitudes,

While the calculated surface current densities are larger at low altitudes than at geosynchronous altitude, the probability that a spacecraft will encounter the hot

R_E	N_- (.../cm^3)	V = 5 keV $\bar{E}_e$ = 1.33 keV $\bar{v}$ = 4.3×10^9	V = 10 keV $\bar{E}_e$ = 2.66 keV $\bar{v}$ = 6.08×10^9	V = 15 keV $\bar{E}_e$ = 3.99 keV $\bar{v}$ = 7.45×10^9	V = 20 keV $\bar{E}_e$ = 5.32 keV $\bar{v}$ = 8.6×10^9	V = 25 keV $\bar{E}_e$ = 6.65 keV $\bar{v}$ = 9.62×10^9
6.6 (24 hr)	6.5	N_- = 4.89 J_- = 3.36	2.44 2.38	1.83 1.94	1.22 1.68	0.97 1.50
6	12.3	N_- = 9.40 J, = 6.47	4.70 4.57	3.13 3.74	2.35 3.23	1.88 2.88
5.5	20	N = 15.0 J_- = 10.35	7.52 7.311	5.00 5.46	3.76 5.17	3.01 4.63
5	36	N_- = 27.1 J_- = 18.6	13.5 13.2	9.00 10.7	6.80 9.29	5.40 8.31
4.5	67.5	N_- = 50.8 J_- = 34.9	25.4 24.f	16.9 211.1	12.7 17.5	10.2 15.6
4.15 (12 hr)	116	N_- = 02.7 J_- = 56.8	41.4 40.2	27.6 32.8	20.7 28.5	16.6 25.5
3.5	313	N_- = 235.0 J, = 161.5	117.5 114.3	78.3 43.3	58.8 00.8	47.0 72.3
v in cm/sec; N_- in particles/cm^3; J_- in namps/cm^2.						

Table 1. Calculated Particle Densities and Initial Primary Current Densities as Functions of Altitude ($\lambda_M = 0^{\circ}$)

plasma at these lbwer altitudes is appreciably less. It would be theoretically possible to calculate an altitude-dependent probability if data were available concerning the probability of encountering a given plasma energy density at geosynchronbus altitude. Such data are becoming available. However, the measured L-dependence of the probability of encountering > 15 volts on an antenna has been measured by IMP-6[3] (see Figure 3). This spacecraft was launched 31 March 1971 into a 28.7° elliptical orbit with an apogee of 32.4 R_E arid a perigee of 243 km. As the data shows, the probability of encountering hot (or at least warm) plasma at 4.15 R_E is approximately an order of magnitude less than at 6.6 R_E. While the inclination of the IMP-6 arbit makes analysis difficult, the data suggest that the spacecraft chafging phenomenum may be more common near geosynchronous altitude than at any other;.

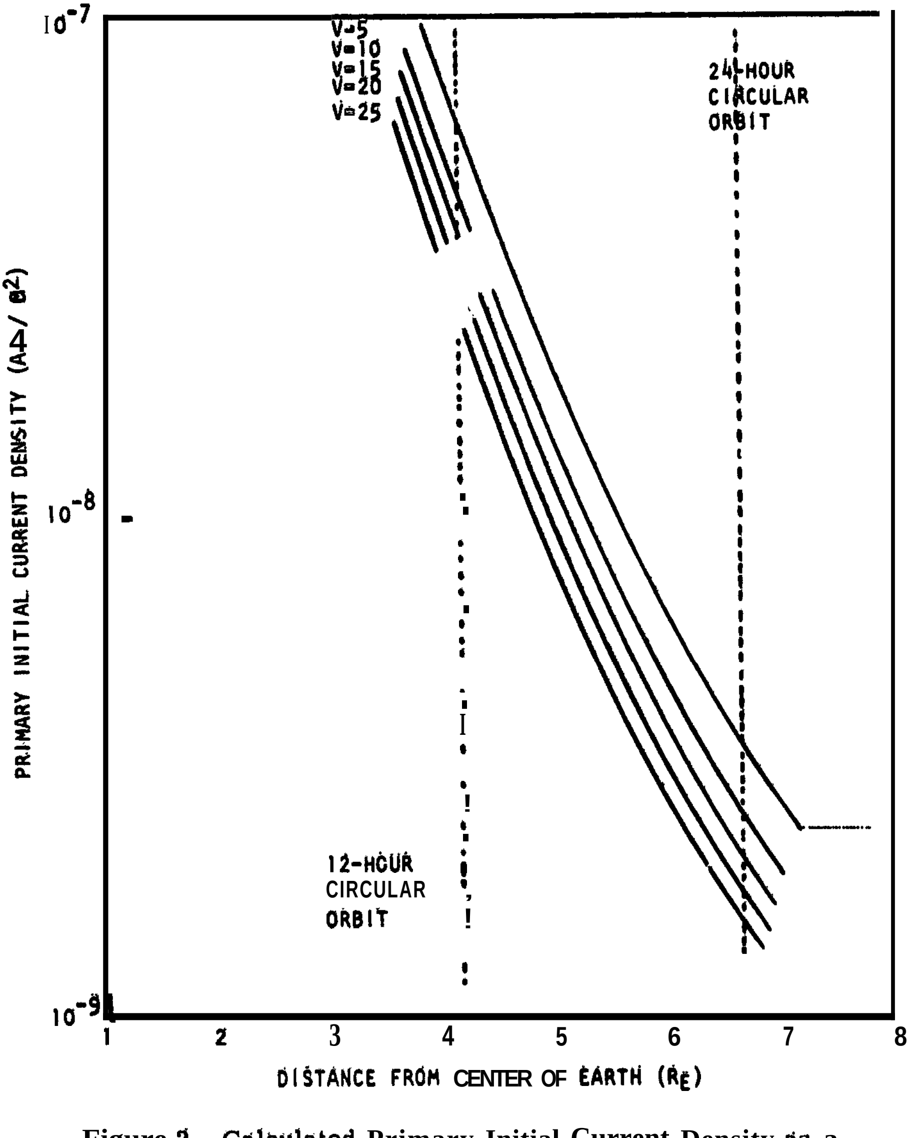

Figure 2. Calculated Primary Initial Current Density as a Function of Altitude ($\lambda_M = 0°$)

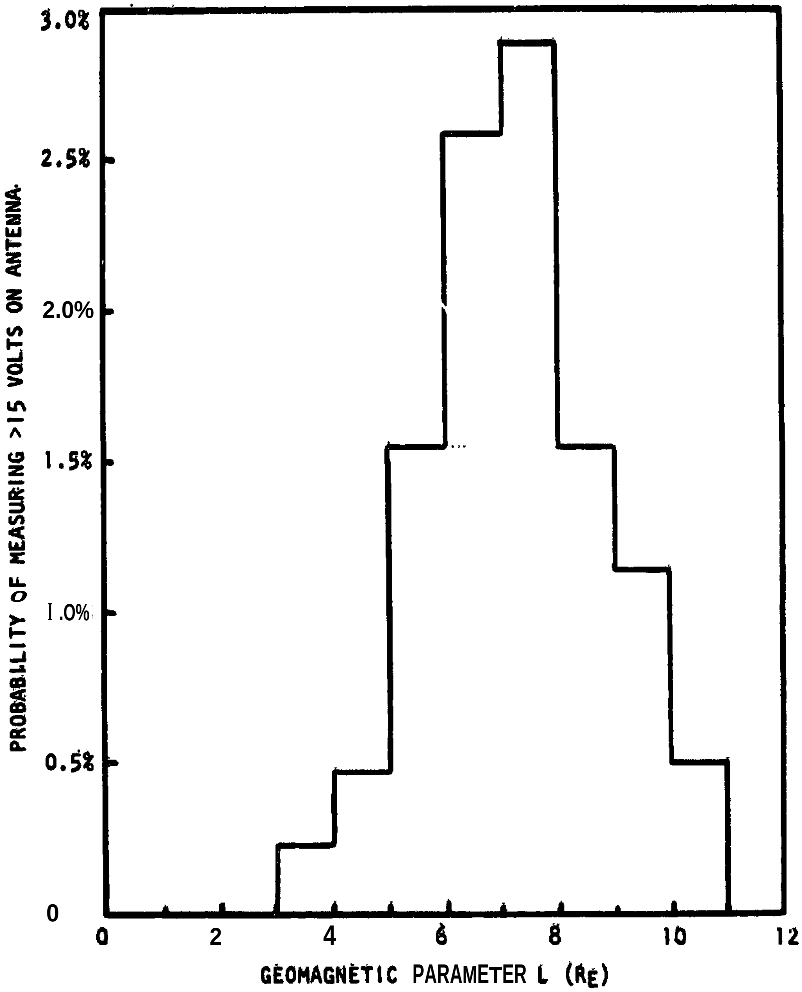

Figure 3. Probability of Measuring > 15 Volts on IMP-6 Antenna

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