

# MEASUREMENT OF CHARGE DISTRIBUTION IN ELECTRON BEAM IRRADIATED PMMA USING ELECTRO-OPTICAL EFFECT

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## Abstract

In space environment, a spacecraft is exposed by high-energy cosmic rays. The electric potential of insulating materials on the surface of the spacecraft charged up when the cosmic rays are irradiated to the spacecraft. Sometimes, the change of the electric potential causes an unexpected accident of electrostatic discharge (ESD) with serious damage to the electric devices. To understand the mechanism of the ESD, the measurement of charge distribution in dielectric materials should be carried out. Therefore, we have proposed to use an optical method using Kerr effect for the measurement. Using this system, we have already succeeded in the measurement of PMMA after the electron beam is irradiated. In this paper, we would like to show the measurement results under the electron beam irradiation in air atmosphere.

## Introduction

The dielectric materials are used for covering the surface of a spacecraft. Since they are exposed to high-energy cosmic rays, the electric potential is increased due to the accumulation of charge in the bulk and/or on surface of them. Sometimes the change of the electric potential causes an unexpected accident of ESD with serious damage to the electric devices. To prevent the accident, it is important to investigate the relationship between irradiation of cosmic rays and dielectric materials. Therefore, we have been developing a measurement system for the charge distribution using an optical technique [1,2]. In this report, the principle of measurement and some typical results are introduced.

## The Principle of the Optical Measurement Method

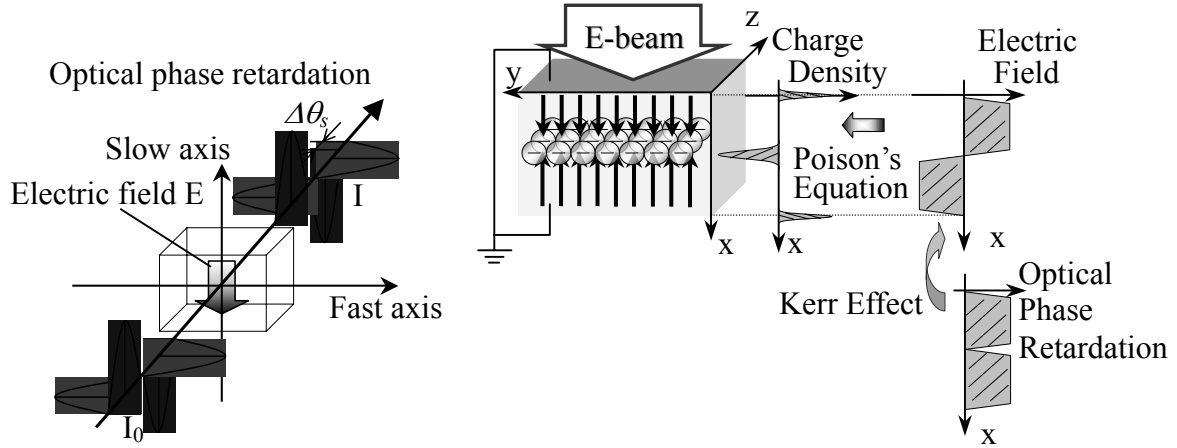
### **Electro-Optical Kerr Effect**

Figure 1 shows the principle of Electro-Optical Kerr effect. When the linearly polarized laser light is illuminated to a sample with electric field  $E$ , an optical phase retardation  $\Delta\theta_s$  generates in the laser light during passing through it. The generated optical phase retardation

$\Delta\theta_s$  is proportional to square of the electric field  $E^2$  as described in following equation,

$$\Delta\theta_s(x) = \frac{2\pi l}{\lambda} \gamma E(x)^2 \quad (1).$$

Where  $l$ ,  $\lambda$  and  $\gamma$  are light path length in the sample, wavelength of the light and Kerr constant, respectively.



**Figure 1. The optical phase retardation  $\Delta\theta_s$  by Kerr effect**

**Figure 2. Model to measure charge distribution using Kerr effect and Poisson's equation**

When the PMMA irradiated with electron beam, electric field  $E(x)$  generated by accumulated charges. Since the birefringence is induced due to the generation of dielectric anisotropy of permittivity, the passing light has the optical phase retardation  $\Delta\theta_s(x)$ . By measuring  $\Delta\theta_s(x)$ , electric field  $E(x)$  is calculated from following equation,

$$E(x) = \pm \sqrt{\frac{\lambda}{2\pi l \gamma} \Delta\theta_s(x)} \quad (2).$$

In the experiments, the wavelength  $\lambda$  is 633nm, distance  $l$  is 10mm and Kerr constant  $\gamma$  is  $1266 \times 10^{-24} \text{ m}^2/\text{V}^2$  [3].

### Poisson's Equation

The charge density  $\rho$  is calculated by the electric field  $E$ . When the charges are assumed to be distribution in  $y$ - $z$  plane as showing Fig.2, the relationship between charge density  $\rho$  and electric field  $E(x)$  is expressed by following equations,

$$\text{div}\mathbf{E} = \frac{\partial E_x(x, y, z)}{\partial x} + \frac{\partial E_y(x, y, z)}{\partial y} + \frac{\partial E_z(x, y, z)}{\partial z} = \frac{\rho}{\epsilon} \quad (3),$$

$$\rho = \epsilon \frac{d_x E(x, y, z)}{dy} \quad (4).$$

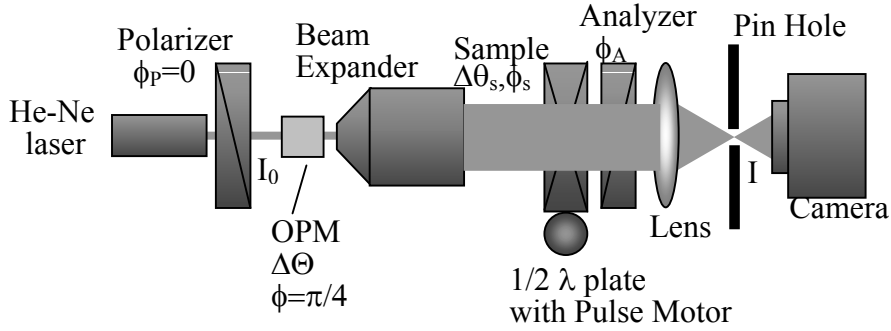
Here, we used the value of 2.68 is used as the relative permittivity of PMMA.

## Measurement System

### Two-Dimensional Birefringence Measurement System

Figure 3 shows a schematic diagram of two-dimensional birefringence measurement system. The optical phase retardation  $\Delta\theta_s(x,y)$  and the fast axis angle  $\phi_s$  of the sample are measured using this system [4,5]. The output light intensity  $I(x,y)$  is given by,

$$I(x,y) = \frac{I_0(x,y)}{2} \left[ \begin{array}{l} 1 + \left\{ \cos 2\phi_A - 2 \sin 2\phi_s \sin 2(\phi_s - \phi_A) \sin^2 \frac{\Delta\theta_s(x,y)}{2} \right\} \cos \Theta \\ - \sin 2(\phi_s - \phi_A) \sin \Delta\theta_s(x,y) \sin \Theta \end{array} \right] \quad (5).$$



**Figure 3. Two-Dimensional Birefringence Measurement System**

Where,  $\phi_A$  and  $\Theta$  are optical rotation angle rotated by  $1/2 \lambda$  plate and phase retardation modulated using BSO crystal, respectively. Modulated light intensities  $I^+(x,y)$ ,  $I^0(x,y)$ , and  $I^-(x,y)$  are described using following equations,

$$I^+(x,y) = \frac{1}{2} \gamma(x,y) I_0(x,y) \left[ \begin{array}{l} 1 + \left\{ \cos 2\phi_A - 2 \sin 2\phi_s \sin 2(\phi_s - \phi_A) \sin^2 \frac{\Delta\theta_s(x,y)}{2} \right\} \cos \Theta \\ - \sin 2(\phi_s - \phi_A) \sin \Delta\theta_s(x,y) \sin \Theta \end{array} \right] \quad (6),$$

$$I^0(x,y) = \frac{1}{2} \gamma(x,y) I_0(x,y) \left[ 1 + \left\{ \cos 2\phi_A - 2 \sin 2\phi_s \sin 2(\phi_s - \phi_A) \sin^2 \frac{\Delta\theta_s(x,y)}{2} \right\} \cos \Theta \right] \quad (7),$$

$$I^-(x,y) = \frac{1}{2} \gamma(x,y) I_0(x,y) \left[ \begin{array}{l} 1 + \left\{ \cos 2\phi_A - 2 \sin 2\phi_s \sin 2(\phi_s - \phi_A) \sin^2 \frac{\Delta\theta_s(x,y)}{2} \right\} \cos \Theta \\ + \sin 2(\phi_s - \phi_A) \sin \Delta\theta_s(x,y) \sin \Theta \end{array} \right] \quad (8).$$

Where  $I_0$  and  $\gamma(x,y)$  are input light intensity and a parameter for the un-uniformity of output light intensity, respectively. From Eqs(6) and (8) the subtraction of  $I^-(x,y)$  from  $I^+(x,y)$  is given as,

$$I^+(x, y) - I^-(x, y) = -I_0(x, y) \sin \Theta \cdot \sin 2(\phi_s - \phi_A) \sin \Delta \theta_s(x, y) \quad (9).$$

Here, we describe  $I_0(x, y) \sin \Theta$  and  $\sin 2(\phi_s - \phi_A) \sin \Delta \theta_s$  as a system function  $I_M(x, y)$  and an intrinsic function  $F(\Delta \theta_s, \phi_s)$ . From Eqs.(6), (7) and (8), the system function  $I_M(x, y)$  and the intrinsic function  $F(\Delta \theta_s, \phi_s)$  are given as,

$$I_M(x, y) = \frac{\sin \Theta}{1 - \cos \Theta} \{I^+(x, y) + I^-(x, y) - 2I^0(x, y) \cos \Theta\} \quad (10),$$

$$F(\Delta \theta_s, \phi_s) = \frac{(I^-(x, y) - I^+(x, y))(1 - \cos \Theta)}{\sin \Theta (I^+(x, y) + I^-(x, y) - 2I^0(x, y) \cos \Theta)} \quad (11).$$

When the optical rotation angle  $\phi_A$  was set  $\pm \pi/8$ rad, the intrinsic function  $F(\Delta \theta_s, \phi_s)$  is given as,

$$F_L(\Delta \theta_s, \phi_s) = \sin 2(\phi_s - \phi_{AL}) \sin \Delta \theta_s(x, y) \quad (12),$$

$$F_R(\Delta \theta_s, \phi_s) = \cos 2(\phi_s - \phi_{AL}) \sin \Delta \theta_s(x, y) \quad (13).$$

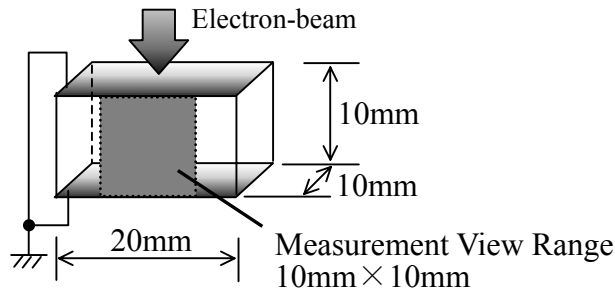
Therefore, using Eqs.(12) and (13), the optical phase retardation  $\Delta \theta_s$  and the fast axis angle  $\phi_s$  are given as,

$$\Delta \theta_s(x, y) = \sin^{-1} \sqrt{F_R(\Delta \theta_s, \phi_s)^2 + F_L(\Delta \theta_s, \phi_s)^2} \quad (14),$$

$$\phi_s = \left( \frac{1}{2} \tan^{-1} \frac{F_L(\Delta \theta_s, \phi_s)}{F_R(\Delta \theta_s, \phi_s)} \right) + \phi_{AL} \quad (15).$$

### **The Electron Beam Irradiation**

When the sample is irradiated with electron beam, the birefringence generates in the sample. The test sample used were a rectangle PMMA with the size of 10mm in height, 20mm in width and 10mm in depth, respectively as shown in Fig.4. Thin conductive layers are put on top and bottom surfaces of the test sample. The electron beam with the current density of  $35 \text{ nA/cm}^2$  was irradiated to the sample with energy of 1.5MeV for 60seconds in air atmosphere. The measurement of the birefringence is carried out during electron beam irradiation and after irradiation.



**Figure 4. The shape of sample and measurement view range**

## **Result and Discussion**

Figure 5 shows two dimensional phase retardation distributions  $\Delta\theta_s(x,y)$  in PMMA during and after electron beam irradiation to the sample. With increase of irradiation time, a dark line that indicates the retardation is nearly zero is getting clear at the depth of ca 3.3mm from the irradiation surface.

Figure 6 shows the time dependent of the retardation distribution around the center of the sample. It is found that the bottom of the curves are located at ca 3.3mm from irradiation surface. Since the sensitivity of the system is too high, some of the value of the retardation  $\Delta\theta_s$  exceeds the measurement limit. On the other hand, the birefringence behavior immediately after the start of the electron beam irradiation is observed because of the high sensitivity of this measurement system. The optical phase retardation  $\Delta\theta_s$  before the electron beam irradiation is due to the originally remaining stress (photoelastic effect).

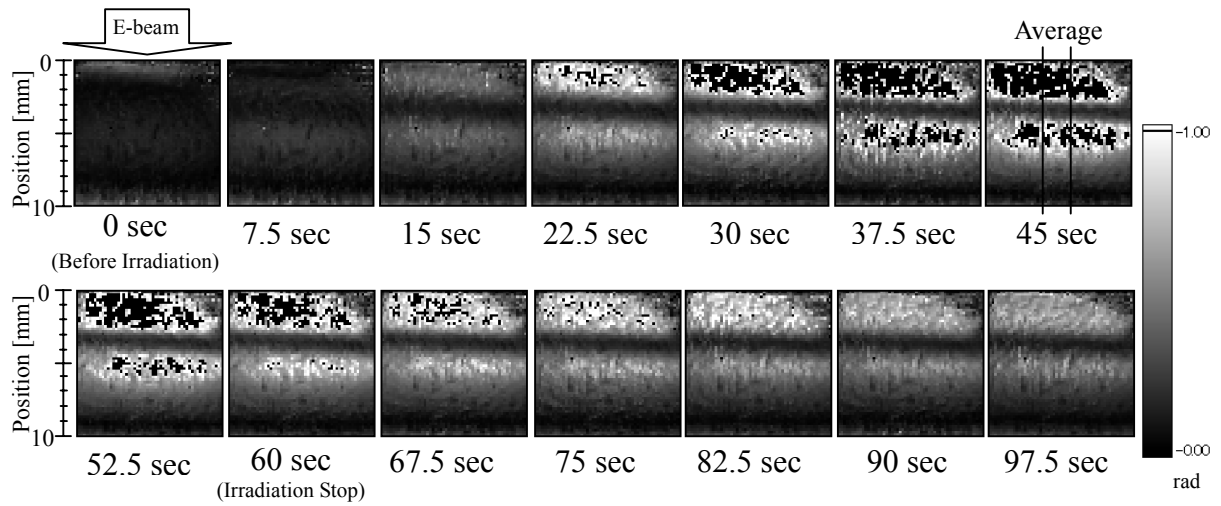
Figure 7 shows the time dependent charge density distribution in PMMA sample during and after electron beam irradiation. The results shown in figure 7 are calculated using equation (1)-(10). It is found that the negative charge is increasing at 3mm depth with increase of electron beam irradiation time. After irradiation, it is found that the accumulated negative charge gradually decreases. Judging from the results, it seems that the electro-optical method is applicable to the measurement of charge distribution in electron beam irradiated sample during irradiation.

## **Conclusion**

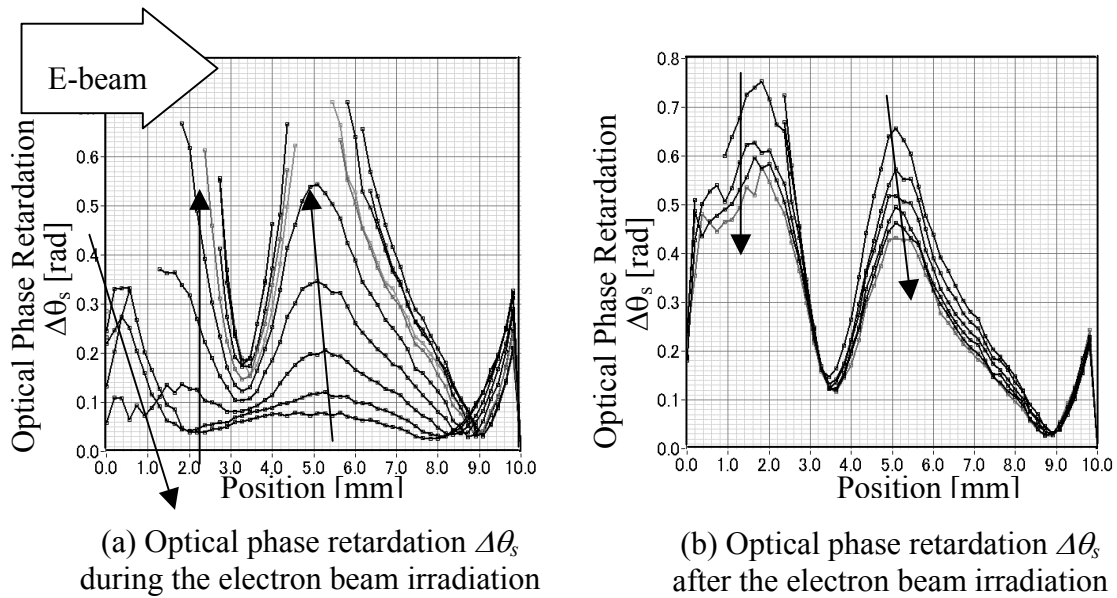
We succeeded in the measuring of the charge density  $\rho$  in PMMA during and after electron beam irradiation. A typical result clearly shows the negative charge accumulation and its decay in electron beam irradiated PMMA sample.

## **Acknowledgement**

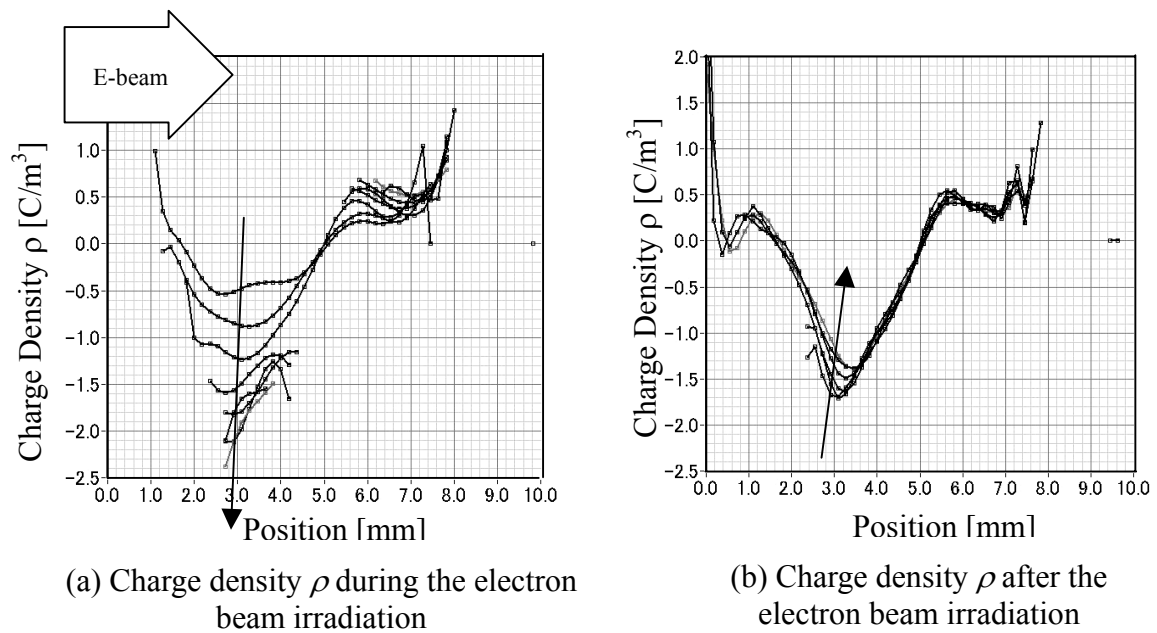
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**Figure 5.** The two-dimensional distribution of retardation in PMMA sample



**Figure 6.** Time dependence of optical phase retardation distribution



**Figure 7.** Charge density calculated using Poisson's equation

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