SPACECRAFT CHARGING IN A QUASI-STATIC AND DYNAMIC PLASMA ENVIRONMENT AND THE SCALING LAWS FOR ESD-INDUCED CURRENT TRANSIENTS

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Objective

The objective of this paper is to present a general expression for predicting characteristics of an electrical pulse from a surface discharge. Scaling laws are presented for various shapes of the dielectric surface.

Abstract

For many years satellite design engineers and manufacturers used a **(**√(*Area*)**)**-scaling law to estimate peak current amplitudes from electrostatic surface discharges (ESDs) on satellites in orbit. Balmain proposed this scaling law (BSL) in 1978. In some applications, Balmain's scaling law generates overly conservative design requirements.

This paper presents a general expression for the pulse shape of surface discharges. For a given surface discharge, the peak current depends on the shape of the surface and on the location of the discharge; the pulse duration depends on the longest linear distance from the site of a discharge to an edge of the surface.

Introduction

Applied physicists always face the challenge of using small-scale laboratory test results to predict what would or could happen to full-scale systems when they are exposed to postulated worst-case environments. Their predictions become the basis for systems design requirements (SDRs). Management must make the difficult decision to implement, or to ignore the recommendations that the technical staff made on the basis of their best technical judgement. Management's concern is that overly conservative requirements drive costs up without corresponding benefit. The alternative of not implementing the recommended design guidelines might result in significant losses from a degraded or lost mission. The economic impact of recent spacecraft anomalies, such as GOES 8, Anik 1, and Anik 2 **[1][2]** reminded the industry that spacecraft charging cannot be ignored: this rekindled interest in spacecraft charging, and revived the search for better ways to prevent ESDs.

In the mid-1970s Balmain **[3]** and others **[4]** studied the problem of surface discharges in the laboratory. Based on his observations, Balmain proposed a **(**√(*Area*)**)**-Scaling Law for estimating peak currents from ESDs **[3]** on satellites in orbit. The Balmain Scaling Law is now widely used throughout the industry, because it is convenient and it is easy to use. However, this convenience can be expensive.

Throughout the industry engineers apply BSL regardless of shape or size of a dielectric surface area. Lack of understanding of the physics of surface discharges and subsequent misapplication of BSL led to unrealistic predictions of thousands of amperes of peak currents from surface ESDs on some spacecraft. Basic physics shows that size and shape of a dielectric surface and the location of a discharge determine the pulse shape and its duration.

Contents

In the first part of this paper we present a brief review of the quasi-static theory of differential surface charging in plasma. We establish the existence of a surface shielding distance, L_B, which explains total-surface and partial-surface discharges by diffusion.

In the second part of this paper we present the complete one-line derivation of the general expression for the pulse shape of a surface discharge. We show that BSL is a special case. Each surface has its own characteristic discharge signature that yields considerable information about the charged surface[1].

Glossary

$kT/q = V_{Plasma}$	is the plasma potential
$K_B = \sqrt{((J\rho/t)/(kT/q))}$	is the inverse of the effective surface shielding distance
$L_B = \sqrt{((kT/q)/(J\rho/t))}$	is the effective surface shielding distance
t	is the thickness of the dielectric film/charge-depth
ρ	is the bulk-resistivity of the dielectric
ρ/t	is the surface resistivity
J	is the net normal-incident plasma current density[2]
$\sigma_q \approx Q/A = [Q/(LW)]$	is the surface charge density

I. Theory of Differential Surface Charging

In a 1987 paper titled " *ASCAT, A Surface Charging Analysis Technique,"* **[5]** the author derived an expression for the differential surface potential across a rectangular dielectric surface of width, $\boldsymbol{W}$, and length, $\boldsymbol{L}$. ASCAT models a rectangular surface that is grounded at both ends ($x = 0, \boldsymbol{L}$).

[1] This observation suggests that surface charging/discharging could be useful as a tool for non-destructive evaluation (NDE) of surface materials.

[2] The components of the net charging current density include the ambient plasma current, J_o=ne$u$, the back-scattered electron current density, J_{bs}, the current density from secondary electrons, J_{se}, the current density from photo-emissions, J_{ph}, and the contributions from positive ions and radicals.

Surface Voltage Profiles

In the ASCAT model the steady-state surface-voltage profile[3] across the dielectric strip is

$$V(x) = \left(\frac{kT}{q}\right)\left(1 - \frac{\cosh\left(K_B\left(x - \left(\tfrac{1}{2}\right)L\right)\right)}{\cosh\left(\left(\tfrac{1}{2}\right)K_B L\right)}\right) \qquad (1)$$

The maximum voltage at the midpoint, where $x = (L/2)$, is:

$$V(L/2)|_{Max} = 2(kT/q)(\sinh(K_B L/4))^2/\cosh(K_B L/2) \qquad (2)$$

__*For* **(*L*) << (*L*$_B$)**__ the maximum voltage is given by: $V_{max} = (1/8)J(\rho/t)\,L^2$. It is half of the incident current, $(1/2)J(LW)$ multiplied into the mean resistance in either grounded half-strip, $R|_{Ave} = (1/2)((\rho(L/2))/(tW))$. We could have used Ohm's Law to derive the same.

__*For* **(*L*) >> (*L*$_B$)**__ the maximum voltage approaches the plasma potential: $V_{Max} = V_{Plasma} = kT/q$. This is a result a physicist would have guessed without doing any analysis at all. It is consistent with the theories of thermodynamics and electrostatics. In thermodynamics we learned that an object immersed in a heat bath will eventually acquire the temperature of the heat bath. In electrostatics we learned that an object suspended in plasma will reach equilibrium at the plasma potential.

Surface Field Distribution

The electrical stress across the dielectric surface is defined by $E(x) = -dV(x)/dx$. It is:

$$E(x) = \left(\frac{kT}{q}\right)\cdot K_B \cdot \frac{\sinh\left(K_B\left(x - \tfrac{1}{2}L\right)\right)}{\cosh\left(K_B\left(\tfrac{1}{2}L\right)\right)} \qquad (3)$$

The maximum stress occurs at both grounded edges:

$$E(x{=}L,0)|_{\mathbf{Max}} = \pm\,(kT/q).K_B.\tanh(K_B\,(L/2)) \qquad (4)$$

__*For moderately conductive surfaces and* ***L* << *L*$_B$**__ this stress is $E(L,0)|_{\mathbf{Max}} = \pm J(\rho/t)(L/2)$.

The maximum stress can also be written as $E(L,0)|_{\mathbf{Max}} = \pm\, 4\,(V_{max}/L) = \pm\, 2\,\mu\, u$, where the mobility, μ, is given by $\mu = (n\, q_e\, \rho) = \sigma_q\,(\rho/t)$, and $u = \langle E\rangle_{\text{Ave}} / \mu$. The factor "2" accounts for the averaging process where $\langle E\rangle_{\text{Ave}} = (E_{max}+E_{min})/2 = E_{max}/2$. The *average* electrical stress can also be calculated from the derivative of the maximum voltage.

[3] The general surface voltage profile is a function of the grounding configuration and the surface resistivity. It is the basis for a grounding guideline that is more effective than the existing multiple-point grounding guideline.

For example, for **(*L*) <<(*L_B*)**:

$$(\mathrm{d}(V_{max})/\mathrm{d}L)|_{\mathrm{Ave}} = \pm(\mathrm{d}/\mathrm{d}L)\ [((1/8)\ J\ (\rho/t)\ L^2)] = \pm(1/4)\ J\ (\rho/t)\ L \qquad (5)$$

As it should be, this is half the value for the maximum electrical stress for **(*L*) <<(L_B)**.

For high-resistant surfaces and **$L >> L_B$** the stress is $E(L,0) = \pm (kT/q)K_B = \pm V_{Plasma}/L_B$. The maximum stress is the slope of the surface voltage profile at the grounded terminals where $x = L$, and $x = 0$. This slope crosses the plateau of the surface voltage at a distance of $x = L_B$ from a reference ground.

Surface Current Distribution

The steady-state current distribution across the surface is given by **[3]**:

$$I(x) = (W \cdot K_B) \cdot \left(\frac{kT}{q}\right) \cdot \left(\frac{t}{\rho}\right) \cdot \left(\frac{\sinh(K_B(x - (1/2)L))}{\cosh(K_B \cdot (1/2)L)}\right) \qquad (6)$$

In the middle of the strip, $I(L/2)=0$. At the grounded edges on either end of the strip (at $x=0$ *and* at $x=L$) the leakage current is given by

$$I(0,L)\big|_{Max} = \mp J \cdot (W \cdot L_B) \cdot \tanh\left(K_B \cdot (1/2)L\right) \qquad (7)$$

For **(L) << (L_B)** $\tanh(K_B L/2) \approx (K_B L/2)$. Therefore, the magnitude of the leakage current at either grounded edge is $|I(0,L)| = (1/2)J(LW)$. This is half the captured plasma current.

For **(L) >> (L_B)** $\tanh(K_B L/2) \approx 1$. Therefore, the magnitude of the leakage current is $|I(0,L)| = J.(W.L_B)$. Note that this leakage current is a local phenomenon, i.e., only that fraction of the total area that is effectively within the surface shielding distance, L_B, from reference ground captures the plasma current. The area beyond L_B is at equilibrium with the plasma potential, kT/q: on those outlying areas, the resident charges shield the surface against incident charges from the plasma.

Total-Surface vs. Partial-Surface Charge (Diffusion) Discharges

The discussion in the previous section suggests that surface discharges on moderately conductive dielectric surfaces[4] where $L<<L_B$ tend to be total-surface discharges. Surface discharges on high-resistant materials where $L>>L_B$ tend to be partial-surface discharges. For the ASCAT model, only that fraction ($f = \# L_B/L$) of the total surface area that falls within several shielding lengths, L_B, will discharge. This is an important concept, because it answers the question about the existence of an upper bound for BSL. It has implications about the total amount of charge (and therefore energy) that can be dissipated in an ESD **[5]**, i.e., *high-resistant dielectrics will not release all surface charges in a single surface discharge*. Since not all of the energy stored on a

[4] "Moderately conductive" means the bulk and/or surface resistivity is sufficiently low that $L<<L_B$.

charged dielectric surface will be dissipated in a single surface discharge, we may conclude that there is justification to relax some ESD design requirements. In Part II of this paper we will explore this issue in further detail from the perspective of the transient nature of an ESD. Let us review the lessons learned until this point.

Lessons Learned

(L#1): For moderately conductive surfaces and $\boldsymbol{L \ll L_B}$ the entire surface area, $A=(LW)$, captures and diverts the plasma current, $I=JA$, to both grounded terminals[5].

(L#2): For high-resistant surfaces and $\boldsymbol{L \gg L_B}$ only a relatively narrow strip of area $A_B=WL_B$ within a few shielding lengths, L_B, from a reference ground captures and diverts the *locally* captured plasma current to the nearest reference ground. The remainder of the surface is at equilibrium with the plasma potential, (kT/q). This surface deflects the incoming plasma current away from the dielectric surface.

(L#3): The concept of a "*locally* captured plasma current," $I_B=JA_B=(L_B/L)(JA)$ is important. It means that around every reference ground there is a strip of land that is as wide as several shielding lengths, L_B. Beyond the borders of this island, surface charges do not move. If the separation distance between two islands is more than 2 L_B, then the charges on one island may not interact with the charges on a second island. Charges on separate islands interact strongly when the borders of two or more islands overlap. We can interpret L_B as the surface analogue of the Debye length in a plasma, L_D.

Theory of Surface Discharges

Surface discharges occur when the electrical stress exceeds the breakdown strength of the medium. There is a general understanding, and laboratory tests confirm it is true **[4]**, that high-resistant materials tend to break down more often than more conductive materials do. This concept is in fact the basis for one of the standard charge mitigation techniques in the industry. Therefore, we limit our discussion to high-resistant dielectrics, and we postulate that breakdown occurs when the electrical stress reaches the breakdown strength of the material. From Equation (4) and $L \gg 2\, L_B$ we can calculate the electrical surface stress for high-resistant materials, and postulate that breakdown occurs if

$$|E(0)|_{\mathrm{Max}} = (kT/q).K_B = \sqrt{((\rho J/t)(kT/q))} \geq E_{BD} \qquad (8)$$

Based on Equation (8) a charge-mitigation design requirement is that the electrical surface tension or stress be kept below the surface dielectric strength of the material. Testing to show compliance takes time and money. Therefore, to cut down on laboratory time, accelerated testing is common practice. Such tests are performed at elevated levels of J and the voltage of the electron beam used to simulate the plasma potential, kT/q. Equation (8) shows that there is an other option to enhance accelerated testing. It is based on the trade-off between two simultaneously occurring effects. (1) For most dielectrics, lowering its temperature increases its

[5] In general, captured charges diffuse to all grounded strips: the longer the grounding strip is, the better the diffusion rate will be, and therefore the more effective the charge mitigation will be.

surface resistivity, thereby accelerating the electrical surface tension to reach the breakdown level **[4]**, and (2) lowering the plasma temperature will lower the electrical surface tension, thereby delaying the electrical surface tension, $|E(0)|_{\mathrm{Max}}$, to reach the breakdown level, E_{BD}. A log-log plot of (ρ/t) vs. (*sample temperature*) shows that the surface resistivity increases with falling temperature. It is a plot often used to determine the activation energy in solids. At phase transitions, such as occurs at the melting point, one observes an abrupt change in the slope, and therefore the activation energy. Test results and on-orbit data showed that some dielectrics, such as nylon, do break down more often at low temperatures. Thus, for the tested samples, the temperature dependence of the surface resistivity, (ρ/t), dominated the observed breakdown. This suggests that testing at lower temperatures is a viable third option for accelerated testing. (One must not confuse the temperature of a sample that affects (ρ/t) with the plasma temperature in kT/q.)

Summary of Part I: Surface Charging Theory

The first part of this paper provided a set of useful equations. In particular,

i) for estimating the maximum steady-state surface potential on dielectric surfaces use Equation (2): $V(L/2)|_{Max} = 2(kT/q)(\sinh(K_B L/4))^2/\cosh(K_B L/2)$

ii) for estimating the maximum electrical stress, use Equation (4): $E(x=L,0)|_{\mathbf{Max}} = \pm\,(kT/q)\,K_B \tanh(K_B\,(L/2))$

iii) for estimating the maximum steady-state diffusion/leakage current at the grounded terminals, use Equation (7): $|I(L,0)|_{\mathrm{Max}} = \pm\, J\,(W\,L_B)\tanh(K_B\,L/2)$

iv) for identifying controllable parameters to accelerate achieving test objectives, use Equation (8): $|E(0)|_{\mathrm{Max}} = (kT/q)\,K_B = \sqrt{((\rho J/t)(kT/q))} \geq E_{BD}$

From "*iv)*" above and the effect of sample temperature on its surface resistivity it is noted that low-temperature testing is a third viable alternative for accelerated testing.

In Part I of this paper we compared the response of moderately conductive materials with the response of high-resistant materials. We showed that for high-resistant materials, only the part of surface area inside a few shielding lengths, L_B, from reference ground captures and diverts the plasma current to reference ground. The remainder of the surface is at equilibrium with the plasma potential, and as such, it serves as a shield against the incoming plasma current. We noted that the shielding length **[5]**, $L_B = \sqrt{((kT/q)/(\rho J/t))}$, is the surface-analogue of the Debye length in a plasma, L_D. It is a shielding distance, i.e., it is the mean free path, or the diffusion length of electrons across the surface of a dielectric material. The shielding length, L_B, is the basis of our postulate that a single electrostatic discharge on high-resistant materials will not dissipate all surface charges and partial-surface discharges will not dissipate all available energy stored on the surface. This may have a significant impact on current ESD mitigation design requirements.

In Part I of this paper we discussed only the quasi-static model of spacecraft charging. Most of the discussion is based on a paper published in 1987 **[5]**. For convenience, we list the most significant results in **Table 1**.

Table 1. Summary of Steady-State Solutions for Surface Charging Effects

Description	**Maximum**	$L \ll L_B$ **:[1A]**•	$L \gg L_B$ **:[fA]**•
Voltage Profile: $V(x)$ = ***Eqn. (1)***	**Eqn. (2):** $V(L/2) = 2(kT/q)\dfrac{(\sinh(K_B L/4))^2}{\cosh(K_B L/2)}$	***V$_{Max}$*:** $V = (1/8)J(\rho/t)\cdot L^2$	***V$_{Max}$*:** $V = V_{Plasma} = kT/q$
Electrical Stress $E(x)$ = ***Eqn. (3)***	**Eqn. (4):** $E(0,L) = \mathrm{m}(kT/q)\cdot K_B \cdot \tanh(K_B \cdot L/2)$	***E$_{Max}$*:** $E = (\frac{1}{2})J(\rho/t)L$	***E$_{Max}$*:** $E = (\frac{kT}{q})/L_B$
Current: $I(x)$ = ***Eqn. (6)***	**Eqn. (7):** $I(0,L) = \mathrm{m}J \cdot W \cdot L_B \cdot \tanh(K_B \cdot L/2)$	***I$_{Max}$*:** $I = (\frac{1}{2})J(LW)$	***I$_{Max}$*:** $I = J(W \cdot L_B)$

- **1A:** Candidate for total-surface discharge
- **fA:** Fractional, or partial-surface discharge if (distance between grounding) > ($2L_B$).

In Part II of this paper we discuss the transient nature of ESD. The objective is to predict the pulse shape, $I(\tau)$, the peak current, $I_{pk} = I(\tau)|_{\max}$, and the pulse width, τ_{pw}, or pulse duration, τ_{Total}, of an ESD. After a brief listing of our assumptions, we will present the complete one-line derivation of the general expression for a surface discharge current.

II. Theory of ESD Transients

Since the early work by Ohm (1827), much research was done on conduction in gasses, metals and dielectrics. (Debye, Thomson, Seebeck, and Lorentz). These and many other authors studied the more complex subjects of how an ESD is triggered and how an arc discharge is sustained: those are subjects beyond the scope of this paper. We only discuss a transient ESD pulse from the time the discharge was triggered until the pulse stops. We derive a general expression for the shape of the pulse, and we apply our expression to representative geometrical shapes. We show that BSL is a special case of a more general scaling law.

Derivation of Scaling Laws

Introductory Remarks In Part-I of this paper we showed that L_B is the surface analogue of the Debye length. L_B is a surface shielding length or diffusion distance. A little later we explain how L_B affects spacecraft mitigation design requirements. However, to keep this simple, we make the following assumptions:

1• A single surface discharge occurred at some point along an edge of a dielectric tile
2• The surface charge density is constant everywhere: $dQ/dA \approx (Q/A) = \sigma_q$ = constant
3•, Regardless its shape, a surface discharge clears the entire surface, A, of all charges
4• The discharge travels across the surface at a constant *radial* velocity, $u = dr/d\tau = r/\tau$

Clarification: These simplifying assumptions *do not negate the facts* that (1) mid-surface discharges do occur, and (2) at the grounded boundaries the surface charge density is not constant. The third assumption implies that $L << L_B$, i.e., we model a surface area that is electrically small. Later we discuss partial-surface discharges on electrically large surfaces where $L >> L_B$. The fourth assumption is justified by virtue of the second and third assumptions. The *average* velocity of a discharge is proportional to the *average* electrostatic stress across the surface of the dielectric material, i.e.: $u = \langle E_{Max} \rangle / (2\sigma_q . \rho / t)$.

Complete Derivation of Surface Discharges

An electrical current is defined as the rate of change of local charges, i.e., $I = dQ/d\tau$. Subject to the above assumptions, this equation can be written in the following form:

$$I(r) = \frac{dQ}{d\tau} = \frac{d(\sigma_q A)}{d\tau} = \sigma_q \cdot \frac{dr}{d\tau} \cdot \frac{dA}{dr} = (\sigma_q \cdot u) \cdot (r \cdot \theta) = \frac{E_{Max}}{2(\rho/t)} \cdot (r \cdot \theta) \qquad \text{(9A)}$$

This completes the derivation of the general expression for the pulse shape from a total-surface discharge.[6] Since $r = u\tau$, equivalent time-dependent expressions are:

$$I(\tau) = \left(\frac{E_{Ave}}{(\rho/t)} \cdot u \right) \cdot \theta \cdot \tau = \left(\frac{1}{\sigma_q} \right) \left(\frac{E_{Ave}}{(\rho/t)} \right)^2 \cdot \theta \cdot \tau = \sigma_q \cdot u^2 \cdot \theta \cdot \tau \qquad \text{(9B)}$$

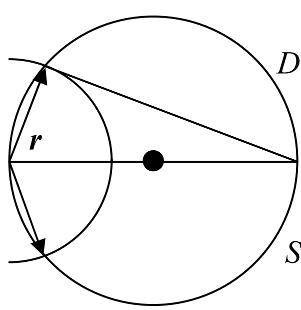
Our studies of surface-ESD transients will be based in part on Equation (9A):

$$\boxed{I(r) = (\sigma_q \cdot u) \cdot (r \cdot \theta) = \frac{E_{Max}}{2(\rho/t)} \cdot (r \cdot \theta)} \qquad \text{(10)}$$

Applications to Specific Geometrical Shapes

Circular tile

The first shape to which we will apply Equation (10) is the circular tile. This is the shape Balmain studied in the 1970's **[6].** His test-results led him to suggest we use the **(**√(*Area*)**)** Scaling Law (BSL) to predict peak ESD amplitudes on large dielectric surfaces.



Disk Geometry

R = Radius of Disk
D = Diameter = 2R
$r\theta$: Arc-length = $2r\,\text{Acos}(r/D)$
$r\theta$: $2r\,Acos(r/D) = 2D\,\theta\,\cos(\theta)$

Surface Discharge Current:

[6] The characteristic pulse shape of natural lightning is remarkably similar. This is not coincidental.

$$I(r) = (\sigma_q u)\left(2 \cdot r \cdot \text{Acos}\left(\frac{r}{D}\right)\right) = 2 \cdot (\sigma_q u)(\theta \cdot \cos(\theta)) \cdot D \qquad (11)$$

The peak current occurs when $dI/d\tau = (dr/d\tau)(d\theta/dr)(dI/d\theta) = u\ d\theta/dr)(dI/d\theta) = 0$:

$$\frac{dI}{d\tau} = 2\sigma_q u^2 \cdot \frac{d\theta}{dr} \cdot (\cos(\theta_x) - \theta_x \cdot \sin(\theta_x)) \cdot D = 0$$

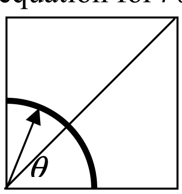
Thus, at $\theta_x = (\cos(\theta_x))/(\sin(\theta_x))$ the peak-current is given by

$$I(r)\big|_{Pk} = 2 \cdot (\sigma_q u)\left(\frac{(\cos(\theta_x))^2}{\sin(\theta_x)}\right) \cdot D \qquad (12)$$

These results show that both the pulse-shape (Eqn. (11)) and the peak-current (Eqn. (12)) scale with $D \sim \sqrt{(Area)}$. ***This is the famous Balmain Scaling Law***. The question is: "Are we doing the right thing in industry when we apply Balmain's Scaling Law to all surface shapes?" To answer that question we next consider a square tile.

Square Tile, dimensions L*W = a*a: Discharge at a corner (0,0)

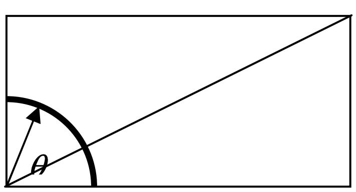
We assume a discharge occurs at a corner. As long as surface charges sustain a spark, we visualize a radial wave front starting at the corner and sweeping across the charged surface. For a square tile, the arc length, $r\theta$, is a piece-wise continuous function of time: it changes abruptly when the wave front meets a corner. For each continuous segment we must write a different equation for $r\theta$.



This diagram shows a square tile of dimensions *"a"-by-"a"*.
For $0<r<a$ $\quad \theta = \pi/2$ radians $\quad I(r) = (\sigma_q . u)\ r\ (\pi/2)$
For $a<r<a\sqrt{2}$ $\quad (\theta) = (Asin(a/r) - Acos(a/r))$ $\quad I(r) = (\sigma_q\ u)\ r\ (\theta)$

For the square tile the peak current is $I(a)|_{Pk} = (\sigma_q\ u)\ a\ (\pi/2)$. This shows that the peak current scales with $a = \sqrt{(Area)}$. This is consistent with Balmain's Scaling Law. However, is this true for other shapes as well? Let's consider a rectangular tile next.

Rectangular Tile, dimensions L*W = 2a*a: Discharge at a corner (0,0)

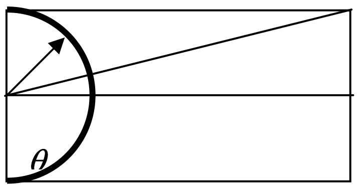


Here we show a rectangular tile of dimensions *"2a"-by-"a"*.
For $0<r<a$ $\quad \theta = \pi/2$ radians $\quad I(r) = (\sigma_q . u)\ r\ (\pi/2)$
For $a<r<2a$ $\quad \theta = Asin(a/r)$ $\quad I(r) = \sigma_q\ u\ r\ Asin(a/r)$
For $2a<r<a\sqrt{5}$ $\quad (\theta) = (Asin(a/r) - Acos(a/r))$ $\quad I(r) = \sigma_q\ u\ r\ (\theta)$

For this rectangular tile the peak current is $I(a)|_{Pk} = (\sigma_q\ u) a (\pi/2)$. Note that its value is *the same* as the value of the peak amplitude for the square tile. It seems that the ambiguity regarding scaling laws for a square tile is removed: On rectangular tiles the peak current scales with the width of the tile. Other researchers have suggested such a scaling law. The total duration of the pulse scales with the longest distance from the arc to an edge of the tile. In this example it is the time to reach the opposite corner. However, most of the surface charge, and therefore surface energy is dissipated after $(L/u) = ((2a) / u)$ seconds.

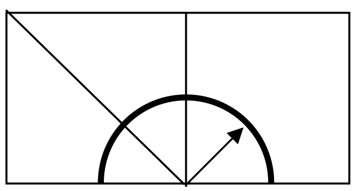
To get a better understanding of the scaling laws we will compare the pulse shape from a surface discharge at half the width, ***W/2***, with one occurring at ***L/2*** $= a$ of the rectangle.

Rectangular Tile, dimensions L*W = 2a*a: Discharge at W/2



The diagram shows the same *"2a"-by-"a"* rectangular tile.
For: $0<r<a/2$ $\theta = \pi$ radians $I(r) = \sigma_q u \pi r$
$a/2<r<2a$ $\theta = Asin(a/r)$ $I(r) = \sigma_q u r Asin(a/r)$
$2a<r<(a/2)\sqrt{(17)}$ $(\theta) = (Asin(a/r) - Acos(a/r))$ $I(r) = \sigma_q u r (\theta)$
The peak current for this configuration is $I_{pk} = \sigma_q u (a/2) \pi$

Rectangular Tile, dimensions L*W = 2a*a: Discharge at L/2



For $0<r<a$, $\theta = \pi$ radians $I(r) = \sigma_q u \pi r$
For $a<r<a\sqrt{2}$, $\theta = (Asin(a/r) - Acos(a/r))$ $I(r) = \sigma_q u r (\theta)$
The peak current for this configuration is $I_{pk} = \sigma_q u (a) \pi$
The calculated peak currents for all five examples are listed in **Table 2**, and correlated with the pulse shapes in **Figure 1.**

If we agree to call the diameter of a circle its width, then we can say that for each shape the peak current scales with the width of the surface. The three examples for the rectangle clearly show that the peak current also depends on the location of the discharge. However, in all cases, the pulse duration for a given discharge is always such that the time integral of the pulse equals the total charge stored on the surface: $\int I\, dt = Q = \sigma_q A$.

This is found to be true for all examples, and the reason is simple: *By our 3rd assumption, we modeled a pulse from a total-surface discharge*. In **Figure 2** we show that for a discharge at a corner of a rectangle, the peak current is not changed as the length of the rectangle is increased. However, the pulse duration increases in such a way that the additional surface under the curve of $I(r=u\tau)$ equals the charge stored on the added surface area. This is the law of conservation of charge, and it is very different from the way we used scaling laws in the past. For a partial-surface discharge, the area that will discharge is within one to seven shielding lengths, L_B, from a site of a discharge.

Putting it all together

Brief summary of accomplishments

The pulse shape of an ESD from a surface discharge is governed by a simple geometrical relationship. For any surface shape of a dielectric tile, the discharge current amplitude is directly proportional to the arc-length of the wave front sweeping across the surface. For the examples in this paper, the center of the traveling wave is located at the site of an ESD[7]. Having said this much, we must be able to qualitatively guess what the pulse shape of a discharge at any point on a dielectric surface of arbitrary shape must look like. We listed the peak-currents for the five examples **Table 2**. The last entry in this table is the universal scaling law for surface discharges. In **Figure 1** we show what each pulse looks like. Equation (9A) shows: $I(r)=(\rho E_{Ave}/t)(r\theta)=(1/2)(\rho E_{Max}/t)(r\theta)= (\textit{Coefficient}) * (r\theta)$. Because it is important, we will say this one more time for emphasis in plain English.

> *The amplitude of a current pulse from a surface discharge to reference ground, measured at the site of the surface discharge, is directly proportional to the arc-length of the wave front that travels across the face of a dielectric surface.*

How to extract Relevant Information from Test Results

In Part II we derived the general expression for the pulse shape for a surface discharge and found that surface discharges scale with the arc-length of a traveling wave front of a discharge. We applied the formula to circular, square, and rectangular shapes, and we showed that Balmain's **(**$\sqrt{(Area)}$**)**-Scaling Law (1978) is a special case: it is the exact solution for circular tiles.

To verify this theory we could perform tests on carefully selected shapes of dielectric material, or we could do a literature search to find what others have done and published since spacecraft charging became an issue. Balmain tested circular samples of varying diameter **[6]**, and so it seems reasonable to say that Balmain had in fact completed the tests for circular samples more than 20 years ago. Therefore, additional testing should be performed on rectangular samples over the temperature range at which satellites might operate. By comparing the measured current pulse shape with predicted pulse shapes one can determine all relevant parameters needed for surface charging assessment purposes. Here is a suggested list of parameters one may want to measure on rectangular samples.

(1) *Average velocity, u, method 1:* From the measured time to reach the first peak and the measured distance between a spark and the nearest corner on a rectangular sample, we can calculate the average speed, u, of a propagating surface discharge. u = (*distance between the site of the spark to the nearest corner, a*) / (*measured time to reach the first peak in the wave shape,* τ) = a/τ.

[7] In all examples in this paper, the arc of the traveling wave front is always in direct line-of-sight with the location of the discharge. For shapes where this is not the case, we may have to use Huygens' Principle of wave propagation to determine the location and the direction of motion of the wave front at any time.

(2) *Average velocity, u, method 2:* An alternative method for finding u is to measure the peak surface potential, and to calculate the theoretical value, $u = \langle E_{Max}\rangle/(2\sigma_q.\rho/t)$, where expressions for E_{Max} are given in Part I, **Table 1**.

(3) *Surface Charge Density, σ_q:* For a corner discharge the peak-current is $Ipk = (\sigma_q.u).(a\,\pi/2)$, and $u = a/\tau$. Therefore, the surface charge density, $(Q/A)=\sigma_q = 2Ipk.\tau/(\pi a^2)$.

(4) *Total Surface Charge:* The total charge is the surface charge density times the area, $Q = \sigma_q A$. We may want to know if the total area, A,[8] is discharged, or only part of it.

(5) From a numerical integration of the ESD pulse we obtain the value for the total charge dissipated in a surface discharge. The ratio between the integrated discharge current and the estimated total surface charge, Q (See #(4) above), is the fraction of surface area that was discharged. This ratio is expected to be of the order of (L_B/L).

Example

Consider a charged rectangular dielectric tile of dimensions $\boldsymbol{W}\text{x}\boldsymbol{L}$ = 1m x 2m, and assume that an ESD occurs at a corner. Assume also that we have a total-surface discharge, i.e., we assume $L_B >> L$. If the measured peak current was 120 amps, and the velocity of propagation was u = 6E5 m/sec, then

A• Estimate the total accumulated charge on the tile just before an ESD is triggered.
B• Estimate or find the pulse duration
C• Estimate or find the average current from a surface discharge

(A•) The third entry in **Table 2** lists the equation for the peak current. It is given by $I_{pk} = \sigma_q.u.(\pi/2)\boldsymbol{W}$. All the parameters needed to calculate the surface charge density, σ_q, are known. Therefore,

$$\sigma_q = (I_{pk})/(\,u(\pi/2)\boldsymbol{W}) = (120)/(6\text{E}5\ (\pi/2)\ 1) = (400/\pi)\mu\text{C/m}^2 = \mathbf{127.3\ \mu C/m^2}$$

The accumulated charge is $Q = \sigma_q A = (I_{pk})\boldsymbol{L}/(\,u(\pi/2)) \approx$ <u>**255 µC**</u>.

(B•) The pulse duration is $(\sqrt{(\boldsymbol{W}^2+\boldsymbol{L}^2)})/u$ = <u>**3.7 µsec**</u>.
<u>Note</u>: One must not confuse the "*Pulse Duration*" with the "*Pulse Width.*"

(C•) The average ESD current is $\langle I\rangle_{\text{Ave}} \approx Q/(Pulse\ Duration)$. Therefore,
$I_{ave} \approx \sigma_q Au/(\sqrt{(\boldsymbol{W}^2+\boldsymbol{L}^2)}) = (I_{pk})\boldsymbol{L}/(\pi/2)/(\sqrt{(\boldsymbol{W}^2+\boldsymbol{L}^2)}) = (120)2/(\pi/2)/(\sqrt{(1+4)})$= <u>**68.33 amps**</u>.
<u>Note:</u> For a long sample, the sustained arc reaches a steady state value given by $I \approx \sigma_q ua = I_{pk}/(\theta) = I_{pk}/(\pi/2)$ = **76.4 amps**.

Summary and Conclusions

The amplitude of the instantaneous discharge current is proportional to the arc length of an expanding spherical wave front of surface discharge: in our examples, the wave front is centered about the site of an arc. In mks-units arc length is measured in (*meters*) x (*radians*). Therefore, a

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[8] How we calculate *Area* is crucial: For a highly resistive material, a recommended estimate is $A=7(L_B)^2$.

universal scaling law for peak currents must be based on a comparison of the maximum arc length, $(r\theta)_{Max}$ of an expanding wave front. Geometry dictates that $(r\theta)_{Max}$ is a function of the shape and size of a charged surface, as well as the location of an arc-discharge. It was shown that Balmain's **(**$\sqrt{(Area)}$**)**-scaling-law is a special case that applies only to circular shapes. One could take liberty to apply Balmain's **(**$\sqrt{(Area)}$**)**-scaling-law to square tiles. However, for all other shapes one must use the universal scaling law: it corresponds to the last entry (in **boldface** font) in **Table 2.**

An important discovery is the fact that the peak amplitude of a surface discharge is less affected by the size of the surface area as it is by the location of the discharge. The pulse duration, however, scales with the distance between the site of a discharge and the most distant point on an edge of the charged dielectric surface.

Recommendations

For the purpose of specifying ESD-induced peak current amplitudes, one must apply the Universal Scaling Law presented in this paper, i.e., $I_{pk} = \sigma_q\, u\, (r\theta)_{Max} = (E_{Ave} / (\rho/t))(r\theta)_{Max.}$ For proper implementation of recommended spacecraft charging mitigation requirements, the Industry needs data for the surface resistivity, (ρ/t), for the operating temperature range for satellites on orbit. This data is needed for estimating $(\sigma_q\, u) = (E_{Ave}/(\rho/t)) = ((E_{Max}/2)/(\rho/t))$ under breakdown conditions.

For partial-surface discharges, one may use **Table 2** to determine the maximum area from which charge is dissipated in a single discharge. **Table 2** shows that beyond (7 L_B), the electrical surface tension, $|E(0)|_{Max}$, is too weak to pull charges into the funnel of a discharge.

Table 2.Percent Surface Discharged as a function of the shielding length, L_B.

(L / L_B)-->>	0.10	0.25	0.50	1.00	3.00	5.00	7.00
Residual Charge	5%	12%	24%	46%	91%	99%	100%
Amount Discharged	95%	88%	76%	54%	9%	1%	0%

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