Electromagnetic Particle Simulation with Unstructured-Grid Model

Masaki Okada, National Institute of Polar Research, Japan
TEL: +81-3-39620-5904, FAX: +81-3-3962-5704, Email: mokada@isc.nipr.ac.jp
Hiroshi Matsumoto, Kyoto Univ., Japan
TEL: +81-774-38-3805, FAX: +81-774-31-8463, Email:matsumot@kurasc.kyoto-u.ac.jp

Abstract:
We have developed an electromagnetic particle simulation code with an unstructred-grid coordinate. This code solves Maxwell's equations which is discretized with triangular elements in 2D simulation space. Plasma particles are also traced by solving the equations of motion with the Buneman-Boris method. The main advantage of this code is the adaptability of modeling more realistic shape of a spacecraft than the orthogonal grid code. Thus, this simulation code is suitable for analyzing the plasma environment in the vicinity of a spacecraft especially in the region within a Debye length from the surface of the spacecraft. We will show the scheme of this code and also show a couple of results from the test simulation runs taking into account of a realistic shape of a spacecraft.

Introduction:
Plasma particle simulations are extensively used by many authors. As an example of the simulation codes, KEMPO (*Matsumoto and Omura* [1984] and *Omura* [1985]) provides a good experimental environment for modeling the fundamental space plasma physics and the non-linear plasma physics. Recent modifications allow us to include the internal boundaries within the experimental space (*Tanaka* [1989]).

Spacecraft design requires more complicated shape to estimate the actual noise level of the scientific instruments. GEOTAIL, for example, is designed as a cylindrical shape. Two masts are extended from the side of the body. The solar panels are also attached on side of the cylindrical body. It was concerned that the noise may be generated by the solar panels because the shadow of the masts cuts the current of the solar panels. Solar Probe is another good example for us to find difficulties on modeling the spacecraft environment. Solar Probe has a heat shield on top of the scientific instruments in order to protect the instruments from the solar radiation. Although we have a couple of choice for the material we are going to use for the heat shield, the cost will highly depends on the material. Decision has to be made on the balance between the cost and the scientific return. A main concern of the engineers is the electromagnetic environment of the scientific instruments at the perihelion ($4R_S$). We have to estimate how it works and how the environment will be modified by the carbon emission from the heat shield. This is the major motivation for developing a new code, which can handle arbitrary shape of a spacecraft.

Algorithms to solve electromagnetic field in the irregular mesh have been introduced by *Seldner et al* [1988]. In order to interpolate field quantities, a two-dimensional area-sharing method was used. The main shortcoming of this method is inefficiency and inaccuracy that arises when mesh with large variations in element size are employed. *Pointon* [1991] introduced a method to handle slanted conducting boundaries. This algorithm can easily be applied to most relativistic electromagnetic particle codes which use the orthogonal grids. Although this method can easily be applied to existing simulation codes which adopt the orthogonal coordinate system, this method can not be applied to a curved boundary.

We adopt triangular coordinate system for the field mesh, both for the electric field and the magnetic fields. This triangular coordinate system can be used for model to fit any shape of the internal and/or the external boundary. A method to solve Poisson's equation in the triangular mesh is well known with the finite element method (FEM). A main shortcoming of this method is that the discretization method of space is not suitable for the time-dependent system. We have developed a new discretization algorithm for the time-dependent triangular coordinate system.

As for a model of particles, *Matsumoto and Kawata* [1990] have introduced a particle-in-cell model using triangular-mesh for the magnetostatic fields. They adopted a circular shape function for the charged particles with a finite radius. Solving the electromagnetic field self-consistently is indispensable for the evaluation of the electromagnetic environment in space plasmas. *Abe et al.* [1986] showed that a higher order spline interpolation removes the limit of the maximum grid spacing relative to the Debye length. The application of the higher order shape function to the triangular grid system is not necessary for the current objectives. For simplicity, we adopt the linear area sharing scheme for the shape function to obtain the charge density and the current density.

Scheme:
Basic equations we solve are the Maxwell equations as shown in equations (1)-(4). Electric and magnetic fields are defined at the staggered time steps with a time difference of $\Delta t/2$, where the Δt is the time step of the simulation.

$$\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t} \tag{1}$$

$$\nabla \times \mathrm{B} = \mu_o \mathrm{J} - \frac{\partial \mathrm{E}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathrm{E} = \frac{\rho}{\varepsilon_o} \tag{3}$$

$$\nabla \cdot \mathrm{B} = 0 \tag{4}$$

The values ε_0 and μ_0 are the dielectric permitivity and the permeability of the free space, respectively. The electric field **E** and the magnetic field **B** are defined as three-dimensional vector space. The conduction current **J** and the charge density ρ are described using plasma particle properties with the following equations.

$$J = \sum_A q_s v_i \tag{5}$$

$$\rho = \sum_A q_s \tag{6}$$

The motion of the charged particles in the electromagnetic fields is described with the equations of motion written as:

$$\frac{dv_i}{dt} = \frac{q_s}{m_s}(E + v_i \times B) \tag{7}$$

Here, the vectors v_i is the velocity of the i-th particle. The quantities q_s and m_s are the charge and the mass of the s-th particle specie, respectively.

In 2D simulation, the simulation space is discretized with the triangular grid. Electric and magnetic field component within XY plane are defined at the side of each triangular element. The Z component is defined at the center of the triangular element. Thus, the rotation of XY component is obtained as a summation of inner product of Exy and the side vector along the triangular element.

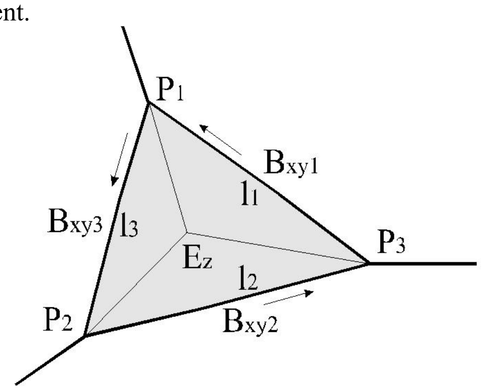

Fig. 1. Electric and magnetic field components within the simulation plane are defined at the vertex of the triangular elements.

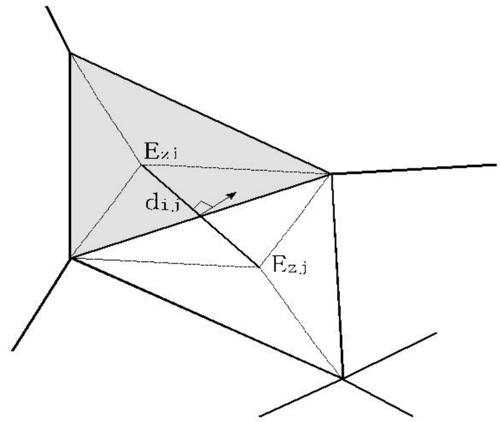

Fig. 2 Electric field and magnetic field components in z direction is defined at the center of the triangular elements.

The rotation of Z component is obtained as the finite difference between two adjacent Z components. Figure 2 shows geometrical configuration between two adjacent triangular elements.

We briefly describe the numerical stability condition of this scheme. The differential equations (1) and (2) are integrated with the leapfrog scheme. We assume **J** = 0 for simplicity. The variable l_k and m_j are the length of the side k and distance between the gravitational centers across the j-th side, respectively. We have

$$B_i^{n+\frac{1}{2}} = B_i^{n-\frac{1}{2}} + \frac{\Delta t}{A_i}\sum_{k=1}^{N_i} E_k^n l_k \tag{8}$$

$$E_j^{n+1} = E_j^n + c^2 \frac{\Delta t}{m_j}\left(B_i^{n+\frac{1}{2}} - B_{ij}\right) \tag{9}$$

$$c\Delta t\left(\frac{1}{A_i}\sum_{k=1}^{N_i}\frac{l_i^k}{m_i^k}\right)^{\frac{1}{2}} < 1 \quad :\mathrm{CFL} \tag{10}$$

Thus, we obtain CFL condition of this code. This indicates that the ratio between the speed of light and the numerical speed must not exceeds unity. This condition must strictly be satisfied for numerical stability. This means that if we use non-uniform grid as a model the smaller time step is required in the smaller grid region. Thus for the particle simulation using leap-frog method uniform triangular grid is more suitable rather than the irregular grid depending on the complexity of the spacecraft. This limitation is not essential as far as we are able to use enough computer resources.

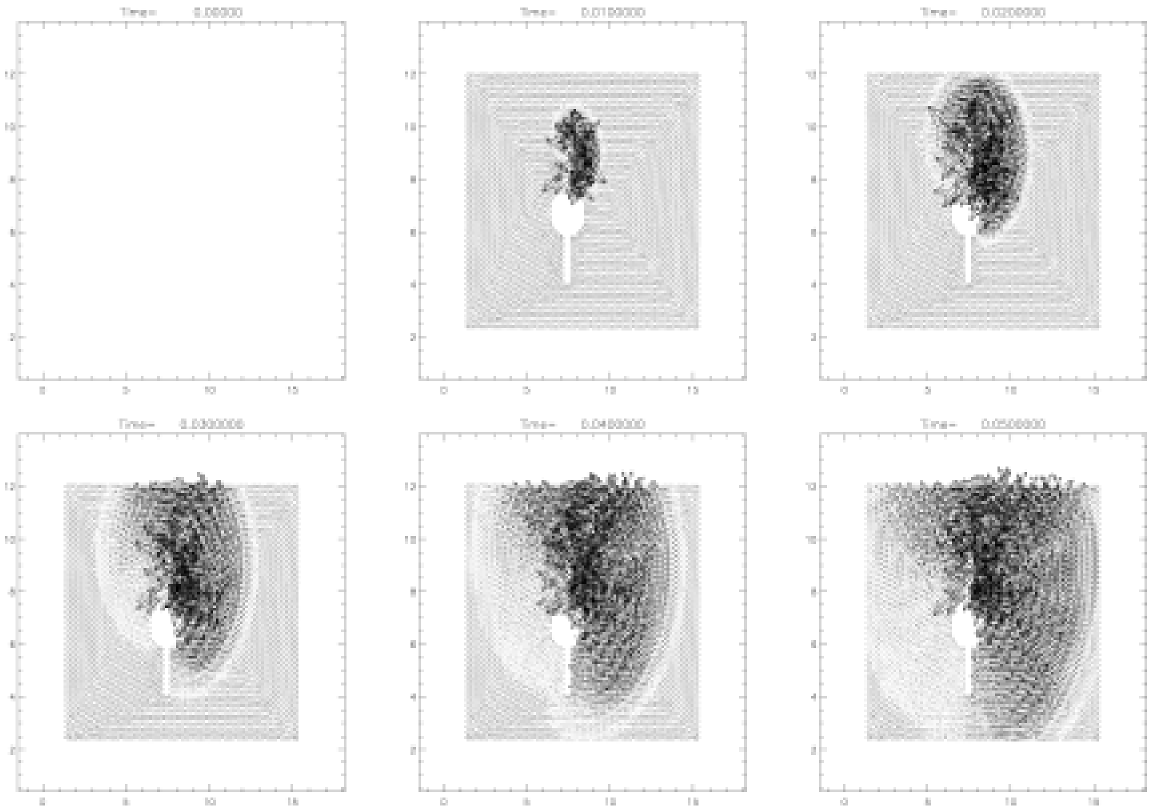

Fig. 3 Electric field vector obtained with a test simulation of the electromagnetic environment around SFU spacecraft. EM field source is located on the right side of the solar panel.

Results:

We have performed two test simulations with different type of internal boundary as a model of a spacecraft. Both results show initial stage of the simulation runs which tests the algorithms for Maxwell equations.

Figure 3 shows the results of a simulation which model Space Flyer Unit (SFU) launched by ISAS, 1996. This spacecraft has two solar panels attached on the main body. This simulation was intended to simulate how the EM wave propagate around the spacecraft due to discharge on the solar panel.

Figure 4 shows another results obtained by modeling a space shuttle like shape as an internal boundary. This simulation run is aimed to solve telecommunication environment of a large spacecraft. Thus the source of the electromagnetic wave is located at the edge of the simulation space.

In both cases, the scheme is properly solved and the electromagnetic environment is clearly reproduced by the simulation. As a test simulation, we adopt fixed boundary condition for the outer boundary.

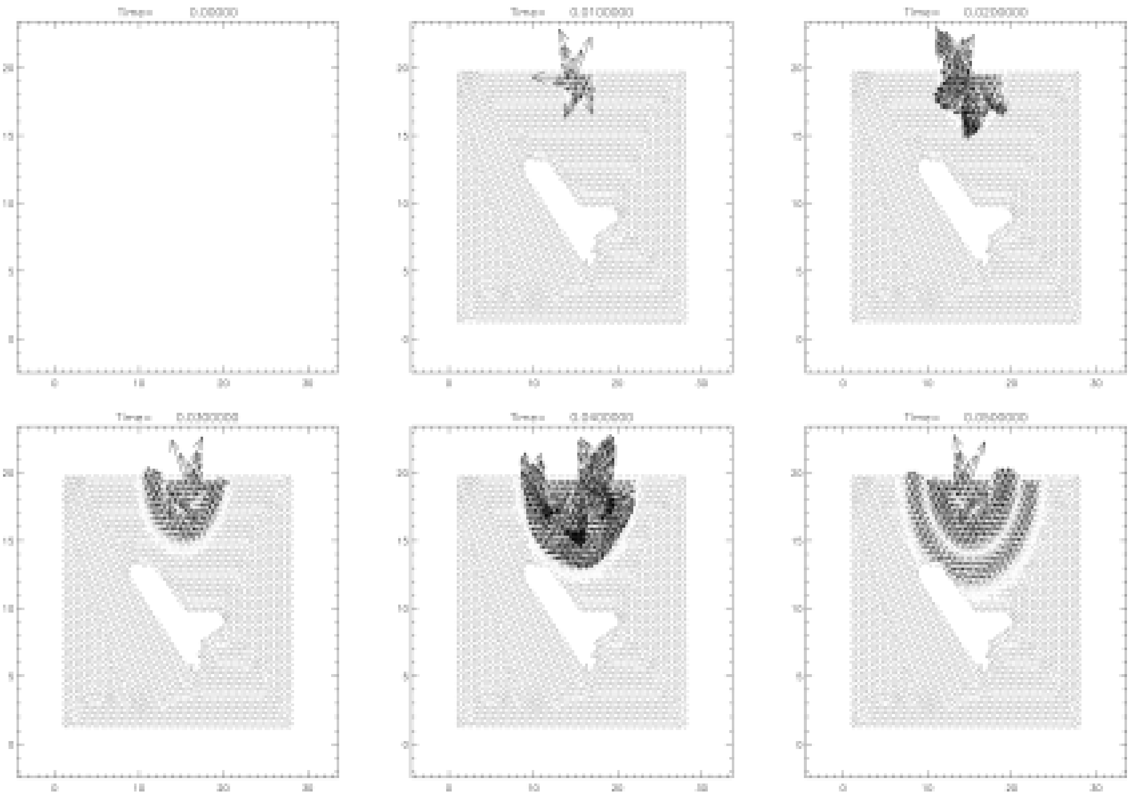

Fig. 4 Electric field vectors obtained with a test simulation of the electromagnetic environment around space shuttle like spacecraft. EM field source is located on the upper edge of the outer boundary.

Summary:

The two-dimensional triangular grid electromagnetic particle code has been programmed and has been evaluated for its performance. We performed two test runs with 4096 grids with two different type of spacecraft as a model. First, the CFL condition has been checked for the light wave without the plasma particles. The energy conservation and Poisson's equation solver have been tested with the test particle simulation.

In order to simulate the charging effect of the spacecraft, charge accumulation has to be taken into account along the internal boundary. 3-dimensionalization and taking into account of material parameters of the spacecraft are left for the future expansion of this code.

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