MAGNETIC SELF-FIELD EFFECTS ON BARE-TETHER CURRENT COLLECTION

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Abstract

It has been recently suggested that the magnetic field created by the current in a bare tether could sensibly reduce its electron collection capability in the magnetised ionosphere, a region of closed magnetic surfaces disconnecting the cylinder from infinity. In this paper, the ohmic voltage drop along the tether is taken into account in considering self-field effects. Separate analyses are carried out for the thrust and power generation and drag modes of operation, which are affected in different ways. In the power generation and drag modes, bias decreases as current increases along the tether, starting at the anodic, positively-biased end (upper end in the usual, eastward-flying spacecraft); in the thrust mode of operation, bias increases as current increases along the tether, starting at the lower end. When the ohmic voltage drop is considered, self-field effects are shown to be weak, in all cases, for tape tethers, and for circular cross-section tethers just conductive in a thin outer layer. Self-field effects might become important, in the drag case only, for tethers with fully conductive cross sections that are unrealistically heavy.

Introduction

Electrodynamic bare tethers work as cylindrical Langmuir probes. Tether bias varies along its length, but typical values of the length-to-thickness ratio are of order 10^6; bare tethers thus collect current per unit length, at each point, as if uniformly polarised at the local bias. An analysis of tether collection requires a detailed theory for the electron-attracting branch of 2D probes with thickness comparable to the electron Debye length λ_{De} and bias highly positive.

The electron current I to a positively polarised cylinder of circular cross section, has an upper bound, I_{OML} (orbital-motion-limited current), which, at high bias, is proportional to the square root of tether bias Φ_P, in addition to being proportional to its length L, cross-section perimeter p, and plasma density N_∞,

$$I_{OML}(p) \approx L \times \frac{p}{\pi} \times e N_\infty \times \sqrt{2e\Phi_P / m_e}\ .$$

This bound is reached for radius less than some maximum value, $R = p / 2\pi \leq R_{max}$, which has been recently determined by way of an asymptotic analysis [J. R. Sanmartín and R. D. Estes, *Physics of Plasmas* **6**, 395 (1999)]. Figure 1 shows general results for R_{max}/λ_{De}. For typical values $T_i \approx T_e$ and $e\Phi_P \approx 10^2$-$10^4\, k\, T_e$ one has $R_{max} \approx \lambda_{De}$.

For $R > R_{max}$ the ratio $I / I_{OML}(p)$ is a function of R / R_{max}, $e\Phi_P / k\, T_e$, and T_i / T_e. This is shown in Fig.2 [R. D. Estes and J. R. Sanmartín, Physics of Plasmas **7**,.4320 (2000)]. The current I is thus, in general, a function of both radius and perimeter.

The above results are valid for an arbitrary convex cross section if the radius R is replaced by some 'equivalent' radius R_{eq}. As examples, one finds $R_{eq} = p / 8$ for a thin tape and $R_{eq} \approx p / 6.78$ for a square cross section. Results are again valid for non-convex cross sections when both radius and perimeter are replaced by equivalent radius R_{eq} and perimeter p_{eq}; for a cross section made of two adjoining circles of radius c, one finds $R_{eq} = \pi c / 2 \approx p_{eq} / 6.55$. For cross sections made of disjoint parts, say two non-adjoining circles, one can still determine equivalent radius R_{eq} and 'effective' perimeter p_{eff}, to be used in the formulae [J. R. Sanmartín and R. D. Estes, *Physics of Plasmas*, to appear].

Self-Field Effects

All the above results apply to unmagnetised plasmas at rest, with unperturbed electron distribution function isotropic, and no trapped-electron population. Tethers, however, move through a plasma at relative speed U_{sat} in the presence of the geomagnetic field B_0. Effects related to both B_0 and U_{sat} (and to trapped electrons) were discussed by Sanmartín and Estes, (1999), and Estes and Sanmartín (2000). There are also preliminary results from laboratory tests and PIC calculations that tentatively indicate that any such effects would not decrease the current collected.

Recently, however, it has been suggested that the magnetic field B_s created by the tether current itself could be so large as to substantially affect the total current to the tether [G. V. Khazanov et al., *J. Geophys. Res.* **105**, 15835 (2000)]. No such effects would influence electron collection by tether-end devices.

Let us first recall the (Parker-Murphy) upper bound that the geomagnetic fielf sets up on the current, in the absence of self-field; at high bias one has

$$I_{PM} \approx \sqrt{\frac{2}{\pi}} \times \frac{l_e}{R} \times I_{OML}$$

where $l_e \equiv v_{th} / \Omega_e$ is a thermal gyroradius, $v_{th} \equiv \sqrt{kT_e/m_e}$ and Ω_e being the electron thermal velocity and gyrofrequency. Clearly, if $l_e >> R$, this upper bound is well above I_{OML}; the geomagnetic field should then have no effect on the current. Actually, the Parker-Murphy bound ignores any space-charge effect; it has been suggested [Sanmartín and Estes, (1999) that an additional condition ($l_e >> \lambda_{De}$) is required for no B_0-effects.

Khazanov *et al*. (2000) modified the Parker-Murphy analysis to take into account the self-field B_s and found a reduction in the PM bound. Further, they gave a rough criterion to determine when the actual current is strongly reduced (say, supressed). The argument involves the new topology of magnetic field lines: there is now a separatrix, with field lines being open outside.the separatrix and closed inside. The characteristic dimension of the separatrix is the radial distance r^* that results from equating fields B_0 and B_s

$$B_s = \frac{I}{2\pi \varepsilon_0 c^2 r}; \qquad (1)$$

with B_0 near perpendicular to the tether one has

$$r^* = \frac{I}{2\pi \varepsilon_0 c^2 B_0}.$$

The simple criterion for full self-field effects, roughly requiring the plasma sheath to lie inside the separatrix, is the condition

$$r_{sh}(\Phi_p) + l_{ef} < a\, r^*(I). \qquad (2)$$

Values 0.3 and 0.5 were suggested for the coefficient a; we shall take $a = 0.5$, which is conservative for our argument that condition (2) does not apply (where it could lead to significant effects on the current). Also, we shall drop, from the condition, the term l_{ef} which represents certain gyroradius for the full field; this is again conservative for our argument. Finally, r_{sh} is a 'sheath radius', which Khazanov *et al* (2000) wrote as

$$r_{sh} \cong \rho_0 \times \sqrt{e\Phi_P / kT_e}$$

with $\rho_0 = \lambda_{De} \times$ a function of R / λ_{De}. Exact calculations in Sanmartín and Estes (1999) and Estes and Sanmartín (2000) show ρ_0 / λ_{De} to be a function of R / λ_{De}, $e\Phi_p / kT_e$, and T_i / T_e,, which is comparable to Khazanov's expression for most values of interest. Condition (2) for strong self-field effects is now

$$r_{sh}(\Phi_p) < a\, r^*(I). \qquad (3)$$

Note that this condition can be applied to, say, tape tethers, Eq.(1) holding at distance $r^* >> R_{eq}$.

Ohmic effects

Use of condition (3) requires knowledge of the relation between current I and bias Φ_p. For current high enough the ohmic voltage drop can not be ignored. Universal relations can be obtained in dimensionless form for all three modes of tether operation by introducing dimesionless current and bias,

$$I \equiv \sigma_c A_c E_m \times i$$

$$\Phi_p \equiv E_m L^* \times \varphi$$

where the motional induced field is typically $E_m \sim 10^2$ V / km and where we introduced a characteristic length for ohmic effects L^*, defined by writing

$$L^* \times \frac{p}{\pi} \times eN_\infty \times \sqrt{2eE_m L^*} \equiv \frac{3}{4}\sigma_c E_m A_c,$$

with A_c the electrically conducting area of the cross section and σ_c the conductivity. Condition (3) now reads

$$\sqrt{\varphi} < C\, i \qquad (4)$$

substantial self-field effects setting up when the ratio $\sqrt{\varphi}/i$, which varies along the tether, drops below a constant C,

$$C \equiv \frac{2^{1/6}\, a}{(3\,\pi)^{1/3}} \left[\frac{B_{\perp}}{B_0} \frac{U_{sat}\, v_{th}^{5}}{c^{6}} \right]^{1/3} \times$$

$$\times \left[\frac{\sigma_c}{\varepsilon_0\, \Omega_e} \right]^{2/3} \left[\frac{A_c^{\,2}\, p}{2\pi^3\, \rho_0^{\,3}\, \lambda_{De}^{\,2}} \right]^{1/3} .$$

with $B_{\perp}$ the component of the geomagnetic field perpendicular to the orbit. Taking $B_{\perp}/B_0 = 0.8$, $U_{sat} =$ 7.5 km/s, $T_e = 0.12$ eV, $B_0 = 0.3$ gauss, $\sigma_c = 3 \times 10^7$ / Ωm, one finds

$$C \approx 2.04 \left(\frac{h}{\lambda_{De}} \right)^{2/3} \times \frac{R_{eq}}{\rho_0} \approx 1.02 \times \left(\frac{h}{\lambda_{De}} \right)^{2/3}$$

for a tape of thickness h. In the final expression we took $T_i \sim T_e$, $R_{eq} \sim R_{max} \sim \lambda_{De}$, and $e\Phi_P /kT_e$ large, leading to $\rho_0 \sim 2\, \lambda_{De}$. Similarly we find

$$C \approx 2.54 \left(\frac{\delta}{\lambda_{De}} \right)^{2/3} \times \frac{R}{\rho_0} \approx 1.27 \times \left(\frac{\delta}{\lambda_{De}} \right)^{2/3}$$

for a circular tether conductive in an outer layer of thickness δ. Also,

$$C \approx 1.60 \left(\frac{R}{\lambda_{De}} \right)^{2/3} \times \frac{R}{\rho_0} \approx 0.8$$

for a fully conductive circular tether. Clearly, C will be small for the first two types of tether.

For the power generation and deboost cases, with the self-field ignored, one has [J. R. Sanmartín, M. Martínez-Sánchez and E. Ahedo, *J. Propul. Power* **9**, 353 (1993)]

$$\sqrt{\varphi} = \left[2 - (i_{max} + i) \right]^{1/3} \left(i_{max} - i \right)^{1/3} . \qquad (5)$$

Power generation, if efficient, will require the current to be a small fraction of the short circuit value, $\sigma_c\, A_c\, E_m$. With i_{max} small, condition (4) leads to a reduced maximum current, $\tilde{i}_{max}$, given by

$$\frac{\sqrt{\varphi}}{i} \to \frac{2^{1/3} (i_{max} - \tilde{i}_{max})^{1/3}}{\tilde{i}_{max}} = C$$

$$\frac{\tilde{i}_{max}}{i_{max}} \approx 1 - \frac{1}{2} i_{max}^{\;2}\, C^3 \approx 1 ,$$

self-field effects being clearly weak for all three types of tethers.

For deboost, maximum current can reach the value $i_{max} = 1$, if the tether is long enough, $L \geq 4\, L^*$ [E. Ahedo and J. R. Sanmartín, to be published]. We would then have

$$\frac{\sqrt{\varphi}}{i} \to \frac{\left(1 - \tilde{i}_{max}\right)^{2/3}}{\tilde{i}_{max}} = C$$

If C is small, we find $\tilde{i}_{max} \approx 1 - C^{3/2} \approx 1$, that is, effects are again weak. However, if C is not small, the case of fully conductive, circular cross sections with R comparable to λ_{De}, $\tilde{i}_{max}$ will be well less than 1. Such tethers would be unrealistically heavy. With the minimum λ_{De} about 3 mm (at N_∞ 10^{12} m^{-3}), taking an aluminum tether, $E_m \sim 135$ V/km, N_∞ 10^{12} m^{-3}, and $R > 3$ mm, $L > 4\, L^*$, one finds a tether mass exceeding 1600 kg.

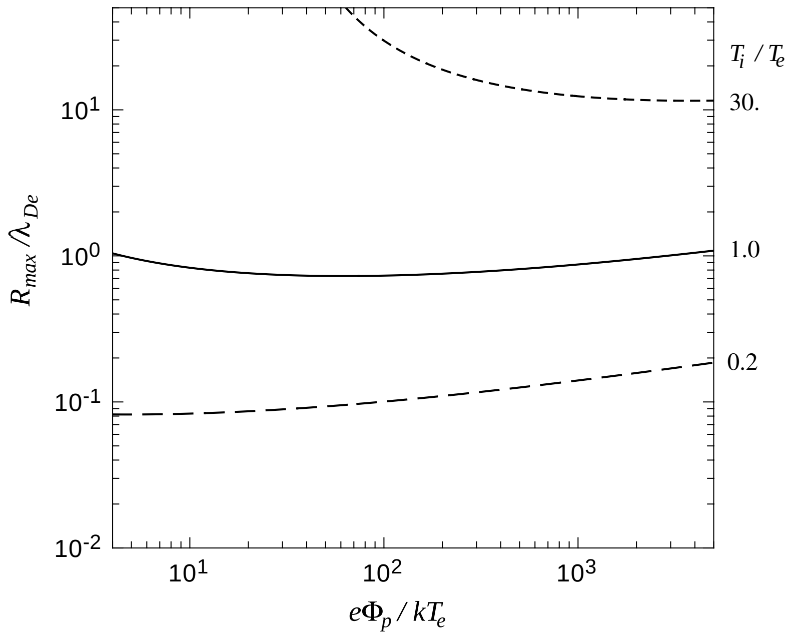
In the reboost case, with the self-field ignored, one has [J. R. Sanmartín, R. D. Estes, and E. Lorenzini, in *Space Technology and Applications International Forum 2001*, ed. M. S. El-Genk, AIP, New York, pp. 479-487 (2001)]

$$\sqrt{\varphi} = (2i + i^2)^{1/3},$$

an expression that does not involve i_{max}. Self-field effects then require a maximum current given by

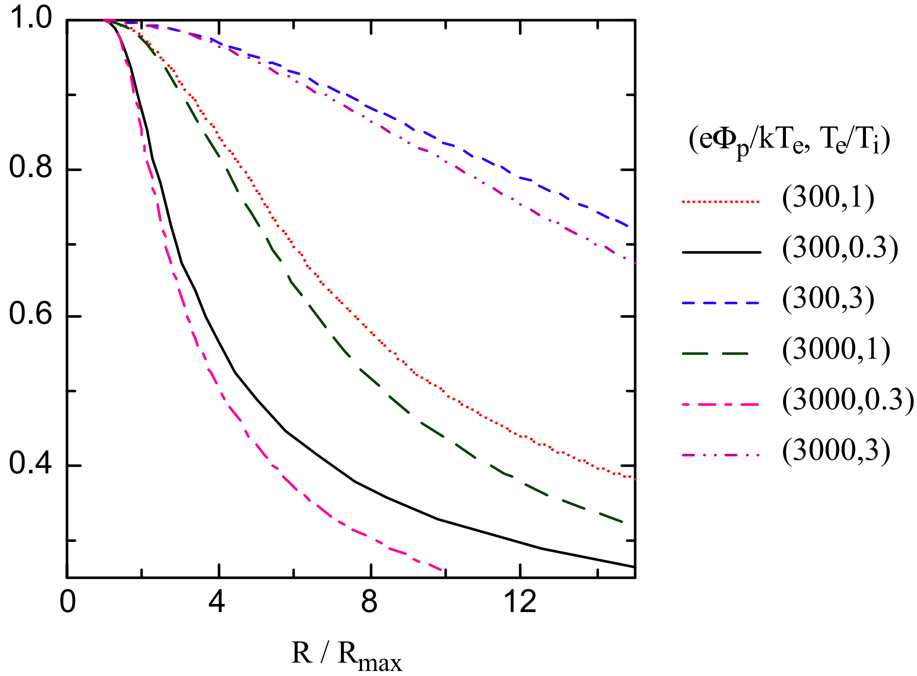
$$\frac{\sqrt{\varphi}}{i} \to \frac{(2 + \tilde{i}_{max})^{1/3}}{\tilde{i}_{max}^{\;2/3}} = C .$$

For C small, self-field effects require i_{max} to exceed $\tilde{i}_{max} \sim 1 / C^3 \gg 1$. For $C \sim 0.8$ self-field effects require i_{max} to exceed $\tilde{i}_{max} \sim 3$. The design parameter i_{max} never reaches such high values, however.



T_i / T_e
30.
1.0
0.2
10^1
10^0
10^{-1}
10^{-2}
R_{max}/λ_{De}
10^1
10^2
10^3
$e\Phi_p / kT_e$

Fig. 1



1.0
0.8
0.6
0.4
0
4
8
12
R / R_{max}
($e\Phi_p/kT_e$, T_e/T_i)
(300,1)
(300,0.3)
(300,3)
(3000,1)
(3000,0.3)
(3000,3)

Fig. 2