### SHORT ELECTRODYNAMIC TETHERS

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#### Abstract

ED bare tethers are best systems to deorbit S/C at end of service. For near polar orbits, usual tethers kept vertical by the gravity gradient, yield too weak magnetic drag. Here we propose keeping tethers perpendicular to the orbital plane. They must be rigid and short for structural reasons, requiring power supply like Ion thrusters. Tether tube-booms that can be rolled up on a drum would lie on each side of the S/C. One boom, carrying an idle Hollow Cathode, collects electrons; the opposite boom's HC ejects electrons. Special HC arrangements to avoid net magnetic torque are discussed. Power source switching must revert the current twice per orbit. Tether orientation is kept Thomson-stable by having the system spin around the tether axis. Also briefly discussed is a second scheme having 4 booms at 90 degrees from each other and rotating fast in an inertial plane perpendicular to Earth's polar axis.

#### Introduction

Electrodynamic bare tethers are the best systems to deorbit S/C in LEO at the end of its operational life. The tether, left uninsulated, collects electrons on some anodic segment. A Hollow-Cathode (HC) plasma contactor ejects electrons at its cathodic end.

For high orbit inclination i, however, the geomagnetic field  $\overline{B}$  lies near the orbital plane: the drag on the usual vertical (long, flexible) tethers might be too weak.

A tether perpendicular to the orbital plane would be better oriented. But the tether length L might then need to be short, drag resulting too weak again.

Here we study this new Short ED tether for high LEO orbits at high i. These short tethers need to beat vertical tethers as well as electrical (Hall, Ion) thrusters. They might result effective for low-mass satellites only.

### **Short Tether Concept**

The tether drag power is:

$$\overline{v}_{sat} \bullet \overline{F} = \overline{v}_{sat} \bullet \left[ L\left(\widetilde{I} \ \overline{u}_{t}\right) \wedge \overline{B} \right] = -E_{m} L \widetilde{I}$$

Where  $\widetilde{I}$  is the average tether current,  $\overline{u}_t$  is an unit vector along the tether making  $\widetilde{I}$  positive, and  $E_m = \overline{u}_t \bullet (\overline{v}_{sat} \wedge \overline{B})$  is the induced electric field  $(E_m > 0 \text{ for drag})$ 

Take a circular orbit, and the tilted  $(\beta_m \approx 11.5^\circ)$  dipole model for  $\overline{B}$ . Then we have:  $E_m = v_{sat} B_{eq} \overline{u_t} \bullet \begin{bmatrix} A_2 \overline{i} - 2 A_1 \overline{j} \end{bmatrix}$ 

$$E_{m} = v_{sat} B_{ea} \overline{u}_{t} \bullet \left[ A_{2} \overline{i} - 2 A_{1} \overline{j} \right]$$

There,  $\bar{i}$  is an upwards unit vector  $\bar{i} \wedge \bar{j} = \bar{v}_{sat}/v_{sat}$ , and  $B_{eq}$  is the field B at the magnetic equator. Also, we have

$$A_1 \equiv \cos\beta_m \sin i \sin\theta + \sin\beta_m (\cos\theta \cos\varphi + \cos i \sin\theta \sin\varphi),$$
  
$$A_2 \equiv \cos\beta_m \cos i - \sin\beta_m \sin i \sin\varphi,$$

where  $\theta$  is the anomaly from the ascending node, and  $d\phi/dt$  arises from both Earth and line-of-nodes

For current driven by the induced bias  $E_m L$ ,  $\overline{u}$ , will oscillate with  $A_1$ ,  $A_2$ , automatically keeping  $E_m > 0$ . In the vertical tether case, we have  $\vec{u}_t = \vec{i} \cdot sign(A_2)$ , yielding  $E_m / v_{sat} B_{eq} = |A_2|$ . For  $I < 78.5^\circ$ , we have  $A_2 > 0$  throughout, and (average)  $E_m / v_{sat} B_{eq} \approx cos$ . Above  $78.5^{\circ}$ , sign( $A_2$ ) oscillates. At  $i = 90^{\circ}$ , the average reaches a minimum  $E_m / v_{sat} B_{eq} \approx sin \beta_m \times 2/\pi$ .

This  $E_m$  reduction directly reduces power drag,  $\overline{V}_{ext} \bullet \overline{F} = -E_m L \widetilde{I}$ . In addition, there is a indirect reduction due to Ohmic effects, from the condition  $\tilde{I}/\sigma A_t < E_m(\sigma)$  is the tether conductivity,  $A_t$  is its cross section area). The drag is dramatically reduced in going from low inclination to near polar orbits.

The optimal inclination would require the tether to rotate in orbit, to keep along  $\vec{v}_{sat} \wedge \vec{B}$ . For high i, however, just try a tether horizontal and perpendicular to the orbital plane,  $\vec{u}_t = \vec{j} \operatorname{sign}(A_1)$ , yielding:  $E_m / v_{sat} B_{eq} = 2 |A_1|$ , average  $E_m / v_{sat} B_{eq} \approx \sin i \times 4 / \pi$ . This is greater than  $\sin \beta_m \times 2 / \pi$  by one order of magnitude.

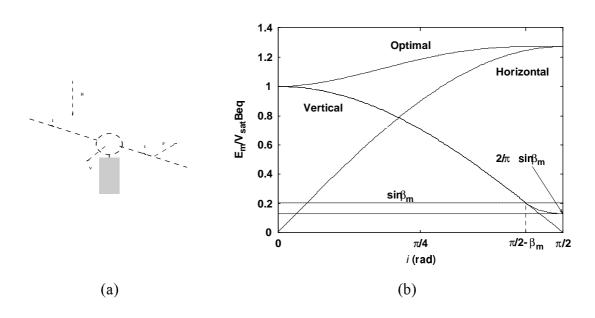


Figure 1: (a) Horizontal tether schematics. (b)  $E_m / v_{sat} B_{eq}$  vs i for tethers horizontal, vertical, and optimally rotating in orbit so as to keep along  $\overline{v}_{sat} \wedge \overline{B}$ 

The gravity gradient at the horizontal orientation is compressive, however. This makes the equilibrium unstable. Also, the tether will need to be rigid and short. This makes it easy to deploy, and free of the flexible-tether instabilities.

Unfortunately, reducing L directly reduces the power drag. It also reduces the bare-tether current  $(I_{BT} \propto L)$ , and the induced voltage  $E_m L$ . A voltage source will be needed to drive the current.

Here we propose using a bare boom of length L/2, with a HC at end, on each side of the S/C. Such booms have been validated in space. They can be rolled up on a drum, and become hollow and rigid when deployed. At every orbital half-period, one boom would collect electrons with its HC off. The HC at the end of the opposite boom would eject an electron current  $I_{hc}$ . A source of voltage  $V_s$  at the S/C is electrically switched twice per orbit. This reverses current, and the way each boom works, to keep  $E_m > 0$ .

Both the Ohmic drop and the ion current to the cathodic boom are negligible. The induced em force  $E_mL$  (~ 40 V) is moderately small against  $V_s$  (~ 200 V). The anodic boom bias is then nearly uniform,  $\Delta V \equiv \Phi_t - \Phi_p \approx V_s$ 

The bare boom current is  $I_{BT} \approx \frac{L}{2} \times \frac{p}{\pi} e \, N_{\infty} \, \sqrt{2 e \, V_{S} \, / m_{e}}$ , where  $N_{\infty}$  is the plasma density, and the cross section perimeter p should not be too large. State-of-art HC's require very little bias,  $I_{hc} \, (\Delta V_{hc}) \approx I_{BT} \, (V_{s})$ , with  $|\Delta V_{hc}| \approx 20 \, \text{V} << V_{s}$ .

The mass flow rate in a  $X_e$  - HC is  $\dot{m}_{hc} \sim I_{hc} \times 1 \text{ sccm / A}$  (1 sccm  $X_e \approx 2.9 \text{ kg / year}$ ). See table.

The current on the anodic boom varies linearly,  $\widetilde{I} = I_{BT} \times \sqrt[3]{4}$ , resulting in a net oscillating torque,  $L^2 I_{BT} / 12 \times [\overline{B} - \overline{j}(\overline{j} \bullet \overline{B})]$ , (~ 3 × 10<sup>3</sup> A m<sup>2</sup>, for  $I_{BT} \sim 1$ A,  $L \sim 200$ m). A magnetotorquer cannot provide a balance. Possible ways to regain equilibrium are:

- i) Set the HC's at a distance  $L/2\sqrt{3}$  from S/C. The ratio  $\widetilde{I}/I_{BT}$  then drops to 0.539.
- ii) Use a 3rd HC at the S/C to eject an electron current  $2 I_{BT}/3$ . The ratio  $\tilde{I}/I_{BT}$  then drops to 0.417.

The resulting equilibrium is still unstable. However, note that the moment of inertia  $I_y$  around the  $\bar{j}$  axis is (much) smaller than moments  $I_x \approx I_z$  around  $\bar{i}$  and  $\bar{v}_{sat}/v_{sat}$  axes. If dissipation is ignored, Thomson equilibrium may then be achieved by means of a spin of angular velovity v around the tether. Thomson stability requires:

$$-3 > 1 + \frac{I_y}{I_x} \frac{\nu}{\Omega} > 3.35$$
 ( $\Omega$  orbital angular velocity).

Dissipation would require imparting a small angular momentum ( $\sim$  3 N×m × minute) a number of times.

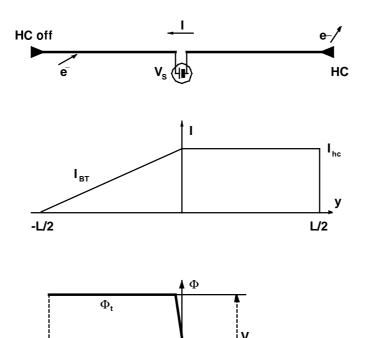


Figure 2: Horizontal tether current and voltage distribution

 $\Phi_{\mathsf{p}}$ 

### Comparison to Alternative Systems

The figure of merit for a thruster is the ratio (dedicated mass  $M_d$ ) / (mission impulse  $F \tau$ ), which should be low.

#### For an ion thruster,

$$\begin{split} M_d &= (1+\alpha) \ \tau \ \dot{m} \ (prop) \ + \ \dot{W} \ (elec.) \ \delta \\ v_{sp} &\equiv F \ / \ \dot{m} \sim 28 \ \text{km/s}; \\ \frac{M_d}{F\tau} &= \frac{1+\alpha}{v_{sp}} + \frac{v_{sp} \ \delta}{2\eta \ \tau} \ \approx \ \frac{1.15}{28 \ km/s} \bigg[ 1 + \frac{1.5 months}{\tau} \bigg] \end{split}$$

#### For a vertical tether,

$$M_d = (1 + \alpha) \dot{m}_{hc} \tau + \alpha_t \times M_{tether}$$
  $(\alpha_t \sim 2.5)$ 

Define  $w = (\rho / \sigma E_m^2) \times F v_{sat} / M_{tether}$ , where  $\rho$  is the tether density, and write:

$$F / \dot{m}_{hc} = \omega_{hc} L \times \widetilde{I} / I_{hc}, \qquad \omega_{hc} \equiv I_{hc} / \dot{m}_{hc} \times B_{eq} \sin \beta_m \times 2/\pi$$

Take a L=10 km, Al tether;  $I_{hc}/\dot{m}_{hc}=1$  A/sccm Xe; 800 km height, and set  $w=\widetilde{I}$  / $I_{hc}=0.75$  (w,  $\widetilde{I}$  / $I_{hc} \rightarrow 1$  at large L).

$$\frac{M_d}{F\tau} \approx \frac{1.15}{230 \, km/s} + \frac{13.3 \, months}{\tau \times 7.5 km/s}$$

A vertical tether needs 4 years to beat an ion thruster.

# For a horizontal tether,

$$M_d = (1 + \alpha) \dot{m}_{hc} \tau + \rho t p L + V_s I_{hc} \delta$$

$$F/\dot{m}_{hc} = \omega_{hc} L \times 3/4, \qquad \omega_{hc} \equiv I_{hc}/\dot{m}_{hc} \times 4 B_{eq}/\pi$$

Structural requirements impose conditions thickness  $t \propto \text{perimeter } p \propto L^2$  (no buckling). We then have:

$$\frac{M_d}{F\tau} = \frac{1+\alpha}{\omega_{hc}L \times 3/4} + \frac{L^5 \times constant}{F\tau} + \frac{4v_{sat}}{3E_mL/V_s} \times \frac{\delta}{\tau}$$

Note that the last term is negligible for multi-month thrusting, as in ion thrusters.

For any given mission impulse,  $M_d/F$   $\tau$  goes then through a minimum at some optimal length  $L_{opt}$ . One readily finds that  $L_{opt} \propto (F\tau)^{1/6}$ , implying that a 2-orders of magnitude jump in  $F\tau$  just needs doubling L. We find that our tether beats the ion truster overn a narrow L range, around 100 m: Too short L makes for too large a  $M_d/F\tau$  ratio; too large L requires too large p, degrading bare-tether collection. At L = 100 m,

$$\left. \frac{M_d}{F\tau} \right|_{\text{minimum}} = \frac{1+\alpha}{\omega_{hc} \, L \times 3/4 \times 5/6} \approx \frac{1.15}{38.4 km/s}$$
 Because  $F\tau \propto L_{opt}^{-6}$ , however, missions with a broad range of total impulse are allowed.

#### **Conclusions**

Bare (boom) tethers perpendicular to the orbital plane are the best systems to deorbit S/C at high inclination. One tube boom (length 75 - 125 m) + HC would lie on each side of the S/C, spinning for Thomson stability. Deorbiting times for a 1000 kg S/C would be 1-2 years except at solar cycle minimum. Structural damping that might break Thomson stability still needs consideration

An alternative scheme would set 4 booms at 90° from each other, rotating in a plane perpendicular to Earth' polar axis. Note that the rotation axis will precess due to the gravity gradient. Rotation much faster than the orbital revolution is required to keep precession down. Switching on a contactor each time its tether becomes anodic might then be hard

## Acknowledgements

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## **Space Plasma Contactors**

(HC types: HC  $\rightarrow$  Ring Cusp, EK  $\rightarrow$  Enclosed Keeper, OK  $\rightarrow$  Open Keeper)

ORIGIN	ACRONYM	EXPELLANT	MASS FLOW RATE (max. readings)	Kg/Year (expellant)	ELECTRIC POWER (Watts)	ТҮРЕ	COMMENTS
USA	SEPAC	Xe	37.4 Sccm	107.6	> 270	RC	Flown on STS-45
66	PMG	Xe	15.0 "	43.17	Batteries	OK	Flown on delta Rocket
66	ISS	Xe	6.0 "	17.27	53.8 peak, 36 running	EK	Planned fur Space Station
EUROPE	STRV	Xe	2.0 Sccm	5.8	< 20	OK	ESA Approved
66	Proel A/300	Xe	0.5 "	1.44	38 peak, 7 running	EK	Engineering model & Testing
66	" A/5000	Xe	3.5 "	10.07	130 " 110 "	EK	cc
66	" A/10000	Xe	5.5 "	15.83	140 " 110 "	EK	cc
RUSSIA	EPICURE	Cs	30 mg/seg	945	2 Kw	ArcJet	Tested in COSMOS flight
66	OKA	Cs	30 mg/seg	945	2Kw	cc	and sounding rocket
66	PROGRESS	Xe	5.5 Sccm	15.6	240 peak, 150 running	EK	
66	GPC-1	Xe, Ar, Kr			Not rated	EK	
66	GPC-2	Xe, Ar, Kr			Not rated	EK	