Debye shielding in a spatially non-uniform plasma: application to plasma wake current collection

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Abstract. The problem of current collection by an electrically biased object in a plasma wake is treated in the context of Debye shielding by a spatially non-uniform plasma. An analytical expression for the decay of the exposed potential in this non-uniform medium is derived, from which an upper bound on collected current as a function of voltage is determined based on the principle of a potential "scoop" pushing its was into the ambient plasma flow. The analytical treatment shows significant agreement to on-orbit current collection data.

Introduction

Laboratory simulations of the plasma wake charging phenomenon[1] have indicated that a negatively biased object in a plasma wake begins to collect significant ion current when its bias is large enough so that the negative potential pushes outside the edge of the shadowing body into the ambient stream, forming a "scoop" from which ions can be collected. The bias level at which this happens represents the limiting potential to which an isolated portion of a spacecraft in low Earth orbit, for example, could charge if exposed to high-energy electrons. Numerical simulations[2-5] also predict this behavior and have successfully used it to predict ion collection data from on-orbit experiments.[6,7] Unfortunately, these numerical simulations are computationally intensive. This paper develops an analytical framework for predicting current collection in plasma wakes based on the general treatment of Debye shielding in a spatially non-uniform plasma.

I. Examining the case of spherical symmetry

Consider a conducting sphere of radius r_0, as shown in Figure 1. If the sphere is biased to a potential Φ_0 and if its surrounding environment is a vacuum, then the potential at any radius $r > r_0$ is given by the familiar solution to Laplace's equation

$$\Phi(r) = \Phi_0 \frac{r_0}{r} \quad . \tag{1}$$

If the sphere is surrounded by a net neutral plasma with equal (unperturbed) electron and ion densities n_0, then the potential is given by the equally familiar relation

$$\Phi(r) = \Phi_0 \frac{r_0}{r} \exp\left[-(r - r_0)/\lambda_D\right] \tag{2}$$

where λ_D is the Debye length, given by $\lambda_D = \sqrt{\varepsilon_0 kT / e^2 n_0}$, where ε_0 is the permittivity of free space, k is Boltzmann's constant, T is the plasma temperature, and e is the elementary charge (this assumes singly-ionized particles).

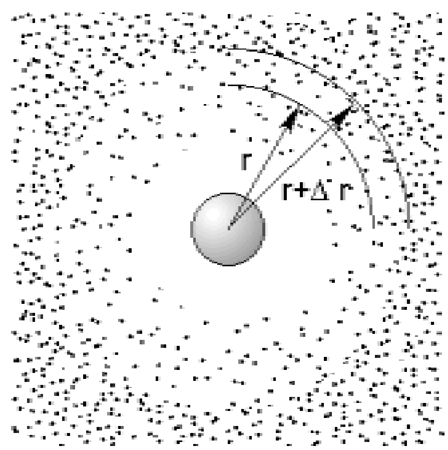

Figure 1. If a radial variation in plasma ion density exists, then the Debye shielding length λ_D will vary with radius as the plasma density n varies, $\lambda_D(r) \propto 1/\sqrt{n(r)}$.

The usual formulation of Debye shielding for a uniform plasma assumes that the ion density is unperturbed by the potential introduced into the plasma, due to the inertia of the ions, while the electrons redistribute themselves according to the Boltzmann relation $n_e(r) = n_0 \exp[e\Phi(r)/kT]$. Now, however, let us assume that the ion density is not uniform, but rather that $n_i = n_i(r)$. We will not, at the moment, describe how such a spatially non-uniform ion density would occur, but rather we will stipulate that it exists. This condition implies that that Debye shielding length varies spatially, $\lambda_D = \lambda_D(r)$, because it varies with the ion density. Let us

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further assume that the length scale of the density variation is large compared to the Debye length at any point, so that Equation (2) is approximately correct for any radius. This implies that if we know the potential at any radius *r*, as in Figure 1, then the potential at a slightly larger radius $r + \Delta r$ is given by

$$\Phi(r+\Delta r) = \Phi(r)\frac{r}{r+\Delta r}\exp[-\Delta r/\lambda_D(r)]. \tag{3}$$

The change in potential $\Delta\Phi$ is given by

$$\begin{aligned}\Delta\Phi &= \Phi(r+\Delta r) - \Phi(r) \\ &= \Phi(r)\left[\left(\frac{r}{r+\Delta r}\right)\exp[-\Delta r/\lambda_D(r)] - 1\right].\end{aligned} \tag{4}$$

In both of these expressions we have taken advantage of the assumption that λ_D varies slowly with radius. Using the expansion $e^{-x} \approx 1 - x + \ldots$, we can write Equation (4) approximately as

$$\Delta\Phi \approx \Phi(r)\left[\left(\frac{r}{r+\Delta r}\right)\left(1 - \frac{\Delta r}{\lambda_D(r)}\right) - 1\right] \tag{5}$$

and keeping only terms that are linear in Δr, we have

$$\Delta\Phi \approx -\Phi(r)\left[\frac{\Delta r}{r} - \frac{\Delta r}{\lambda_D(r)}\right]. \tag{6}$$

Taking the limit as $\Delta r \to 0$, we get the differential equation that describes the potential, namely

$$\frac{d\Phi}{dr} = -\Phi\left[\frac{1}{r} + \frac{1}{\lambda_D(r)}\right]. \tag{7}$$

The general solution to this differential equation is

$$\Phi(r) = \frac{C}{r}\exp\left[-\frac{dr}{\lambda_D(r)}\right] \tag{8}$$

where the constant C is determined by the boundary condition that $\Phi(r_0) = \Phi_0$. In the limit that the plasma density $n_0 \to 0$, the Debye shielding length $\lambda_D \to \infty$, and Equation (8) becomes identical to Equation (1). For the case of a constant plasma density (and hence a constant Debye length), Equation (8) becomes identical to Equation (2). For any other spatial distribution of ions, if $n_i(r)$ is known, the potential distribution $\Phi(r)$ can be determined.

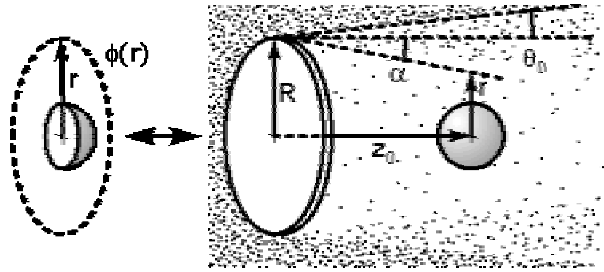

Figure 2. An object moving through a plasma at greater than the ion acoustic speed creates an ion wake. The wake region is a region where there is a strong spatial variation in ion density. An applied negative potential on a body in the wake will tend to collect ions as the influence of the potential pushes into the ambient flowing plasma, forming a "scoop" for ambient ions. Because the extent of the potential's excursion into the wake is greatest at the axial location of the biased body, we can use the results for the spherically symmetric case, in which $\lambda_D(r) \propto 1/\sqrt{n(r)}$, to describe the electrostatic potential as a function of radius at the location of the biased body $z = z_0$.

II. Choosing a distribution function for ions

In the previous section, we assumed a non-uniform distribution in the density of ions without regard to how that distribution might arise. There is, in fact, a situation in which a highly non-uniform distribution of plasma ions can be readily created. If an object, many Debye lengths in dimension, moves through a plasma at a speed that is greater than the ion sound speed, this object will create a plasma wake structure downstream of its location. Orbital dynamics and the parameters of the plasma environment in space near the earth dictate that any object in low Earth orbit creates such a plasma wake. Objects in LEO orbits are termed mesosonic because they are moving at supersonic speeds through the in fluid, and subsonic speeds through the electron fluid. This means that negatively-charged particles have ready access to structures in the wake that are electrically isolated from the spacecraft frame, and can initiate charging events. Spacecraft in highly-inclined orbits are especially vulnerable to wake charging events since they are exposed to high-energy electrons in the auroral zones.

The structure of the ion wake is formed as ions from the ambient plasma fill in the void in the wake of the moving object. Hastings[8] has described the process in terms of an ion acoustic shock, analogous to a supersonic shock in the atmosphere, giving the density of ions in the wake as

$$n(\alpha) = n_0 \exp\left[-\sqrt{M_0^2 - 1}(\theta_0 + \alpha) - \frac{(\theta_0+\alpha)^2}{2}\right] \tag{9}$$

where M_0 is the Mach number of the plasma flow, given by the ratio of the flow speed v to the ion acoustic speed c_s, and θ_0 is the angle defined by the edge of the expansion fan as it propagates into the ambient plasma, given by $\theta_0 = \tan^{-1}(1/M_0)$. Here the angular coordinate α is measured from the edge of the structure forming the wake, as shown in Figure 2. This expression can be simplified in the limit of high Mach numbers, and expressed as a function of the downstream distance z and the radial distance r as

$$n(r,z) = n_0 \exp\left[-1 - M_0 \frac{(R-r)}{z}\right] \tag{10}$$

where R is the distance from the center of the shadowed object to the edge of the object forming the wake. This expression is more tractable than Equation (9) when the time comes to use it in Equation (8).

Equation (9) or (10) describes a model that is non-physical for the deep wake, because it yields a non-zero ion density even for the case of $\alpha = 90$, which would imply an infinite radial impulse applied to some of the plasma ions. More detailed modeling and simulation of the plasma expansion into the wake [9-12] indicate that there is a sharp discontinuity at the leading edge of the expansion front, ahead of which the ion density is negligible. Nevertheless, as Figure 3 shows, the plasma density predicted by Equation (9) or (10), and its effect on the space potential, is limited to the near vicinity of the ambient flow, and Equation (10) is a tractable form for use in modeling the space potential distribution.

If we assume that a circular object forms the wake, the structure of a plasma wake will be cylindrically symmetric, while Equation (8) is based on an assumption of spherical symmetry. Nevertheless, we will fix the axial distance $z = z_0$ in Equation (10) and apply Equation (8) to a cross-sectional slice through the potential structure in the wake region. This approach is justified for several reasons: 1) we are interested in the cross-section that the potential structure presents to the flowing plasma, and both laboratory and numerical simulations[1-5] indicate that this is greatest at the z-position of the biased object, 2) laboratory simulations[1] have shown that the potential structure surrounding a biased object in the wake approximates spherical symmetry to lowest order, and 3) straightforward analytical approximations verify these assertions—for example, the difference in the space potential between that due to a biased sphere alone and that due to a dipole formed between that same sphere and a grounded plate is less than 5% along the cross-section indicated in Figure 2, for $(z_0/r_0) > 5$. With $z = z_0$ and the density given then as a function of r alone, we can calculate the Debye length as a function of radius,

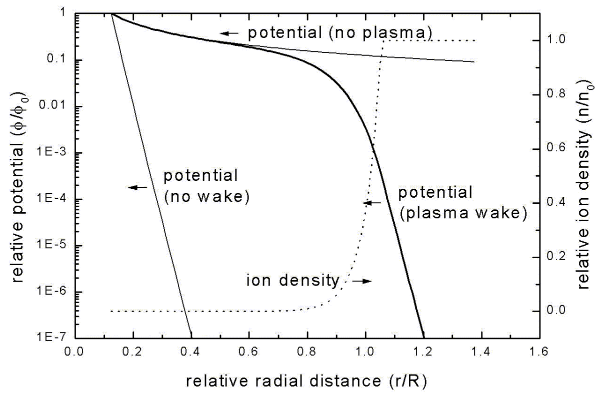

Figure 3. Since the ion density in the deep wake is low, the potential initially falls off following the solution for a potential in a vacuum. Nearer the wake edge, where the plasma density begins to increase, the potential falls off faster with radius, falling off ever more rapidly as the plasma density approaches the ambient level.

$$\begin{aligned}\lambda_D(r) &= \sqrt{\frac{\varepsilon_0 kT}{e^2}}\frac{1}{\sqrt{n(r)}} \\ &= \sqrt{\frac{\varepsilon_0 kT}{e^2 n_0}}\sqrt{\exp\left[1 + \frac{M_0(R-r)}{z_0}\right]} \\ &= \lambda_{D0} \exp\left[\frac{1}{2} + \frac{M_0(R-r)}{2z_0}\right]\end{aligned} \tag{11}$$

where $\lambda_{D0} = \sqrt{\varepsilon_0 kT / e^2 n_0}$ is the Debye length in the unperturbed plasma, far from the influence of the shadowing body. Using this result in Equation (8), we find that the potential $\Phi(r)$ as a function of radius is given by

$$\Phi(r) = \frac{C}{r}\exp\left\{-\frac{2z_0}{M_0 \lambda_{D0}}\exp\left[-\frac{1}{2} - \frac{M_0(R-r)}{2z_0}\right]\right\}. \tag{12}$$

The constant of integration C can be determined from the knowledge of the potential on the biased object, $\Phi(r_0) = \Phi_0$, so that the final result is

$$\begin{aligned}\Phi(r) = \Phi_0 \exp&\left\{-\frac{2z_0}{M_0 \lambda_{D0}}\exp\left[-\frac{1}{2} - \frac{M_0(R-r_0)}{2z_0}\right]\right\} \\ &\cdot \frac{r_0}{r}\exp\left\{-\frac{2z_0}{M_0 \lambda_{D0}}\exp\left[-\frac{1}{2} - \frac{M_0(R-r)}{2z_0}\right]\right\}\end{aligned}. \tag{13}$$

Figure 3 shows a calculation of the potential versus radius for a particular geometry and set of plasma parameters. The plasma parameters, temperature $kT = 0.1\,\text{eV}$, density

$n_0 = 10^{11}\ \mathrm{m}^{-3}$, ion atomic mass $\mu = 16$, and flow velocity $v = 7\times10^3\ \mathrm{m/s}$, were chosen to represent the plasma encountered by a satellite in low Earth orbit. These plasma parameters give a Mach number for the flow of $M_0 = 9.0$.The geometry used for the calculation, radius of the biased object $r_0 = 0.05\ \mathrm{m}$, downstream distance $z_0 = 0.2\ \mathrm{m}$, and distance to the edge of the shadowing body $R = 0.4\ \mathrm{m}$, are representative of the Charging Hazards and Wake Studies (CHAWS) experiment flown aboard Space Shuttle missions STS-60 and STS-69, which investigated current collection by a highly biased object in a plasma wake.[6,7] For comparison, the Figure shows the potential as a function of radius for no plasma present, and for a biased object surrounded completely by plasma at the ambient density. As shown in the Figure, the plasma has little effect on the potential distribution near the biased object, because both the ion and electron densities are low. Essentially, in the deep wake, there is no plasma, and the model predicts that the space potential follows the vacuum solution. Only near the wake edge, where the plasma density becomes significant, is the potential significantly shielded.

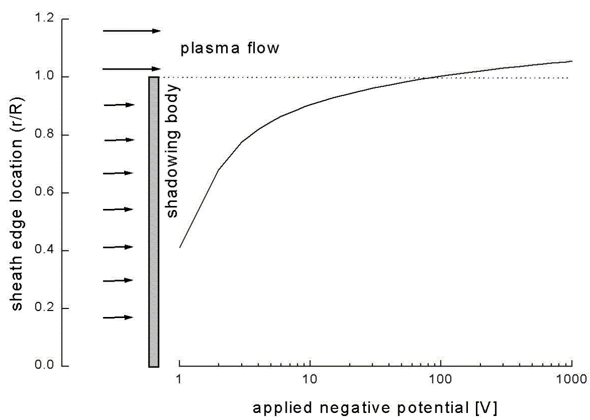

Figure 4. Defining the edge of the current-collecting "scoop" as the point at which the applied negative potential in the wake falls off to the level of $|\phi(r)| = 3kT$, we see that the location of the limit of the influence of the applied bias is a strong function of the applied bias, in particular expanding rapidly at the onset of the bias potential.

III. Potential structure and current collection—a "scoop" model

The extent to which the potential due to the biased object can penetrate the ambient plasma determines the amount of ion current that the biased object can collect. For the application of low Earth orbit, it is this ion current that balances the current collected due to energetic electrons in the environment, and thus establishes the limiting potential to which an electrically isolated object in the wake can charge. If the charging is from an artificial source, this ion current represents a load on the power supply and a possible source of damage due to sputtering.

Because the potential structure pushes out of the shadow of the object forming the wake into the ambient, we may think of it as forming a "scoop" that draws ions into the influence of the biased object. Because both the wake edge and the edge of the potential structure are "fuzzy", it is important to define, or at least approximate, the point at which the influence of the applied potential becomes negligible. Since 95% of particles in a Maxwellian distribution have an energy less than 3*kT*, a conservative estimate is that when the magnitude of the potential Φ falls below this value, the influence of the applied potential will drop off rapidly. Using this definition, and the plasma parameters used in the previous calculation, the location of the edge of the "scoop" is shown in Figure 4. The Figure shows that initially, the influence of the applied potential expands rapidly with increasing bias potential, although the growth of the "scoop" slows as the potential structure encounters the denser plasma near the wake edge.

Using this criterion, we can calculate the ion current collected by the biased object as a function of its potential Φ_0. The first step is to determine the radius of the edge r_e of the scoop by solving Equation (13) so that

$$\Phi(r_e) = 3kT\ . \tag{14}$$

Equation (13) has no closed form solution for r_e, but a numerical solution is easily found. The ion current is then proportional to the flux of ions that enter the "scoop," and is given by

$$I(\Phi) = \int_{r_0}^{r_e(\Phi)} evn(r)2\pi r dr\ . \tag{15}$$

where $n(r)$ is given by Equation (10) with $z = z_0$.

Figure 5 shows the results of such a calculation using the same parameters as previously. Figure 5 also shows, for comparison, the on-orbit results from the CHAWS flight experiment in the presence of similar plasma parameters.[7] The analytical results and the flight data share the same salient features—a rapid onset of current at applied potentials on the order of a few volts, and fully developed current collection on the order of microAmperes. The analytical and experimental data diverge at approximately 40 V. This inflection point in the flight data is usually attributed to the onset of current enhancement due to secondary electron emission that is not accounted for in the analytical model. Nevertheless, the agreement between experiment and theory is striking.

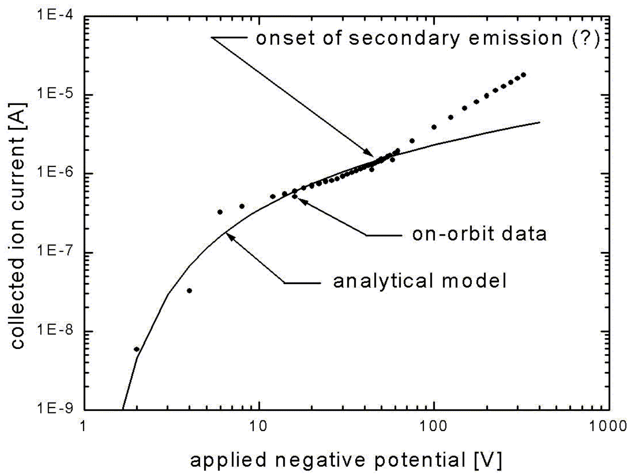

Figure 5. The collection of ion current by an applied negative potential increases as the potential increases, as the influence of the potential pushes into the ambient plasma. The predictions of this analytical approach agree significantly with the results of wake ion collection experiments in low earth orbit.

In fact, one would expect this model to predict an upper bound on the collected current significantly larger than that observed on orbit, because it implies that every particle entering the "scoop" is collected by the biased body. If particles stream into the scoop with a high *z*-velocity, they still face an angular momentum barrier impeding their collection. One possible explanation is that turbulence in the plasma—evidence of which has been seen on orbit[6] and in the laboratory[13]—reduces the particles' angular momentum with respect to the biased object. Another possible explanation is the presence of low-Mach-number hydrogen in the plasma stream.[2, 4, 5] The presence of such a constituent is well established in general, however, it is impossible to determine whether it is present in the CHAWS environments due to the presence of the turbulent component and shifts in the spacecraft potential, and therefore to determine which of these competing explanations is responsible for the CHAWS current-voltage behavior.

Conclusions

The motion of an object through a plasma at greater than the ion acoustic speed creates a spatially non-uniform ion distribution in the plasma wake, implying a spatial variation in the ability of the plasma to shield electrostatic potentials. This variation in Debye length strongly affects how applied potentials penetrate into the non-uniform plasma. If the ion spatial distribution is known, the space potential can be calculated. Negative applied potentials will tend to attract ions from the plasma, as the penetrating potential "scoops" ions from the flow. The collected current calculated using this approach agrees will with experimental measurements of ion collection in plasma wakes by highly biased objects.

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