Effect of the magnetic field on current balance between two conductors in space *

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ABSTRACT

Effects of the magnetic field on the interaction between two spherical conductors in a space plasma is studied. A semi-analytic model is developed and used to investigate the mechanisms by which the flux of ambient electrons and photoelectrons "guided" along the magnetic field lines may strongly affect the current balance between two conductors. Only the particular situation when the system is used as a Langmuir probe to determine the plasma density is studied in more details. The conditions under which the parasitic photo-electron current overcome the ambient plasma current are determined. A critical parameter for this is the temperature of the ambient plasma.

Key words: Space instrument; Langmuir probe; current collection; magnetic field.

1. Introduction

Electrostatic probes are commonly used in space plasma for measurement of low-energy charged particle density, temperature, and ambient electric field. The principle is relatively easy: one exposes a conductor (called a probe) to the plasma and monitors the current of the probe and its potential relative to a reference conductor (See Figure 1). The potential-current relation depends on the plasma properties. Its derivation, however, may be extremely complicated in some situations. For instance, it is usually assumed that the plasma is Maxwellian, and that the space charge and, on high altitude orbits, the magnetic field can be neglected. Then the electron current collected by a spherical probe is given by *Mott-Smith and Langmuir* [1926],

$$I = 2\pi r_p^2 n_e q \sqrt{\frac{k_B T_e}{2\pi m}} \left(1 + \frac{q\phi_p}{k_B T_e}\right) \qquad (1)$$

As shown below, the magnetic field cannot be neglected in the case of low altitude polar orbits. An example is provided by the scientific Swedish satellite Viking launched on 22 February, 1986, into a polar orbit. Its plasma instruments included two Langmuir's probes positioned at the extremities of two 40 meters wire booms in the plan of spin.

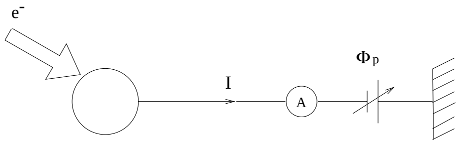

Figure 1. Langmuir probe system.

A periodic oscillation of the current was occasionally observed with twice the spin frequency. An example is given in Figure 2. Between 2034:45 and 2035:20, the Viking satellite crossed an auroral arc. Oscillations with one order of magnitude amplitude are then observed with a maximum corresponding to the time when the probe-hub axis was parallel to the magnetic field line. Outside this region, oscillations with an amplitude of about a factor 2 are observed. These oscillations are minimum when the probe-hub axis was parallel to the magnetic field line. The phase of both types of oscillation indicated that they are not due to the variation of the current of photoelectrons emission with the orientation of the satellite with respect to the sun.

The variation of the probe current observed during the auroral arc crossing was interpreted by the variation of the amount of the photo-electrons that are generated along the boom and confined to a narrow region by the magnetic field. The maximum of this current corresponds to the position when the boom is parallel to the magnetic field [*Hilgers et al.* 1992]. Another effect, competing with the previous one and apparently at work at the edge of the arc is the decrease of the probe current due to the fact that the probe is in a kind of magnetic wake "behind" the spacecraft when the boom is parallel to the magnetic field. The problem is to understand why the first effect is predominantly observed inside the arc whereas the other one is dominant outside it.

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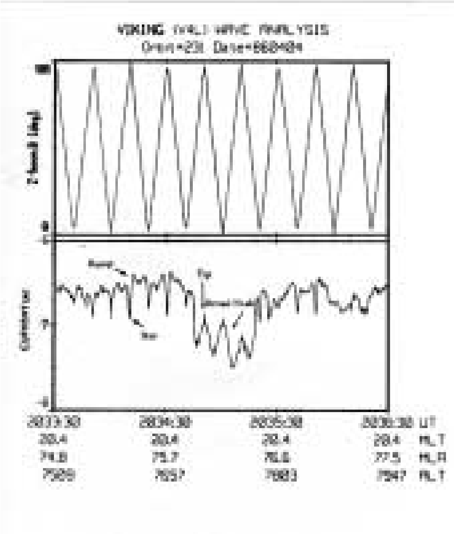

Figure 2. Probe current (lower panel) during Viking orbit 231 and the corresponding inclination of the boom with respect to the magnetic field (upper panel)

The respective magnitude of these two effects have been studied by *Hilgers* [1995] via an analytical approach for which several crude approximations were made. In particular, the various integrals over the electron velocity distribution functions were replaced by mono-kinetik terms. The purpose of this report is to re-analyse the problem without making such simplifications.

2. Model

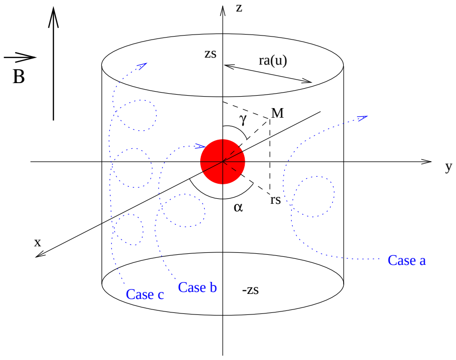

Figure 3. Collection radius : The particles coming from the lateral surface of the cylinder cannot reach the probe (case a). The particles coming from the top or the bottom can reach (case b) or not (case c) but are all considered as reaching the probe. The trajectories are an artist's view.

According to *Rubinstein and Laframboise* [1982] and *Hilger et al.* [1992], particles with speed u and at a distance larger than a collection radius ra,

$$ra(u) = \frac{u}{\omega} + \left[\frac{u^2}{\omega^2} + r_p^2\left(1 + \sqrt{\frac{8(\frac{mu^2}{2} + q\phi_p)}{mr_p^2\omega^2}}\right)\right]^{\frac{1}{2}} \tag{2}$$

cannot reach the probe. Therefore, an upperbound of the total current to the probe is given by the particles coming from the top and bottom surface of a cylinder with basis of radius ra and parallel to the magnetic field. One can further define [cf *Hilgers* 1995] an average coefficient $P(u)$, or collection factor, such that the effectively collected current is

$$\xi = qn_e 2\sqrt{\pi}\int_0^\infty P(u)ra(u)^2\left(\frac{u}{u_{Th}}\right)^3 e^{-\left(\frac{u}{u_{Th}}\right)^2} du \tag{3}$$

Using the property that the exponential polynomial term of the distribution function has a strong maximum for a velocity $u_e^* = \sqrt{\frac{3k_bT_e}{m}}$ one derives an approximation of the current,

$$\xi^* = 2\pi r_a(u^\star)^2 P(u^\star) J$$

where J is the thermal electron current.

Such approximations were made by *Hilgers* [1995] for all types of integrals appearing in the current balance equation of the Langmuir probe system without any further justifications. In the following some assesments of the validity and the consequences of such approximations are provided.

3. Collection factor P(u)

In order to carry on analytical computation of the interactions between two conductors it is convenient to know the values of P as a function of u. Using the above approach and identifying the exact current ξ^* with the current I_{SL}, computed by *Sonmor and Laframboise* [1993], a first-order estimate of P is

$$P(u^*) = \frac{2i_{SL}r_p^2}{ra(u^*)^2}$$

where i_{SL} is the tabulated dimensionless current as given by *Sonmor and Laframboise*, taken for $k_BT = \frac{1}{3}mu^{*2}$.

In such a way, it is possible to derive from the integral values of the current a valud of the collection factor for all velocity. In the following this method is referred to as the method of the peak.

In Figure 4 the values of $P(u)$ computed with the method of the peak is shown for different values of the probe potentials with respect to the plasma. This plot shows that $P(u)$ is only weakly depending of

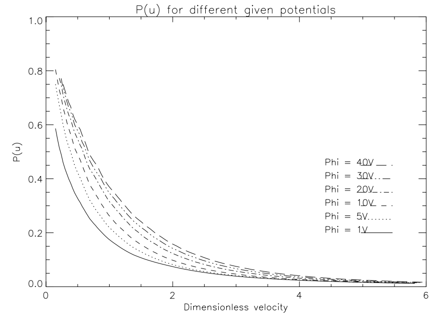

Figure 4. Values of $P(u)$ computed withe the method of the peak for different values of the probe potentials with respect to the plasma.

the potential. This property was an implicit assumpion of the paper by [*Hilgers* [1995]. In the same way it was found that $P(u)$ depends only weakly on the magnetic field.

In order to check the accuracy of the peak method $P(u)$ has been computed from the inversion of the linear system constituted by the discretisation of integral values of ξ as given by equation [3].

The values of $P(u)$ given by the two methods are compared in Figure 5. The two computations appear to be in very good agreement.

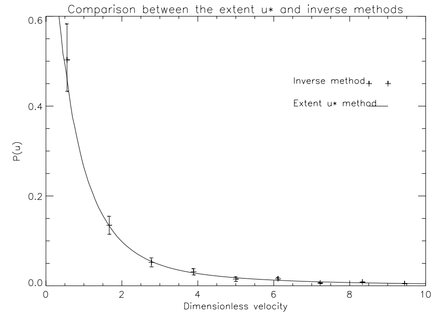

Figure 5. Comparison between the values of $P(u)$ computed on the one hand by the peak method (solid line) and the on the other hand by the inversion method (dots).

4. Double probe current system

The satellite-probe system is sketched in Figure 6. It is modelled by two spherical Langmuir probes aligned along the direction of the magnetic field. Let conductor 1 and conductor 2 be respectively the probe and the hub or the boom of the satellite. For each species of particles i and for each conductor j there is a corresponding collection surface S_{ij} corresponding to the disc of radius r_a. Wherever an interaction occur between the domain of collection (or emission) of the conductors the intersection of the two discs define an area Σ_i.

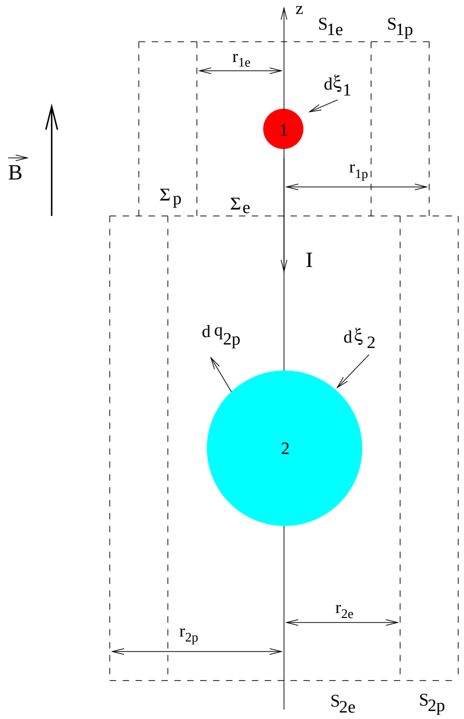

Figure 6. Geometric description of the global system.

In figure 6, the symbol $d\xi_i$ stands for the elementary ambient electrons current and dq_{pi} for the elementary photo-electrons current. Since the probe is biased at high positive potential compared to the photo-electrons typical energy, the flux of photo-electrons coming from the probe can be neglected. Hence, the current balance equations can be written [*Hilgers* 1995].

$$\int (1-\frac{2AP_{1p}\Sigma_p}{S_{2p}})\frac{S_{2p}dj_{2p}}{2A} = \int (P_{1e}S_{1e}+P_{2e}S_{2e}-2P_{1e}P_{2e}\Sigma_e)dj$$

and

$$I = \int P_{1e}(S_{1e} - P_{2e}\Sigma_e)dj + \int P_{1p}\Sigma_p dj_{2p}$$

The above system of equation must be solved to derive the floating potential ϕ_2 and the probe current I.

In the above referred work the system was simplified by replacing in the integrals the non-exponential polynomial terms by their values for which the exponential polynomial term were maximum. Hence, the integration was trivial provided a value of P was determined for instance by the method described in section 3. The results were presented according to the sign of the parameter i,

$$i \simeq \frac{I - P^*_{1e}S^*_{1e}J}{P^*_{1e}S^*_{1e}J}$$

which is the normalized current in excess compared to the "non-perturbed" case. If i is positive the perturbation is of the type observed in the aurora (photoeletron bump) whereas if i is negative the perturbation is of the type observed outside the aurora (magnetic wake).

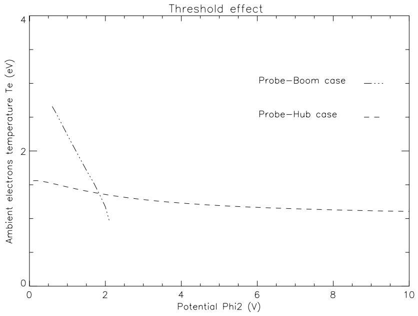

Figure 7. Estimate of the threshold for the ambient electron temperature using the mono-kinetic terms to compute the integrals.

The results derived by using the mono-kintik terms to approximate the integrals is shown in Figure 7 [*Hilgers* 1995]. Two different cases have been considered. In the case of a conductor 2 with a size large compared to the larmor radius of the photo-electrons, it was found that at a constant potential, there exists a threshold value for the electron temperature T_e, T_0, such that for any $T_e > T_0$, the current collected by probe 1 is enhanced by the photoelectrons emitted by the conductor 2. However, if the conductor 2 has a size small compared to the photo-electron Larmor radius the threshold becomes a strong varying function of the potential and therefore both a change in temperature and a change in the ground potential may affect the sign of the modulation. It is believed that this last behavior may match the one of the boom.

In figure 8 the domain of sign of i but based on the full integration of the current balance equation without approximation by mono-kinetik terms. A very similar behaviour as the one previously derived is found apart from a smoother variation of the threshold curve in the case of the small size photo-electron source.

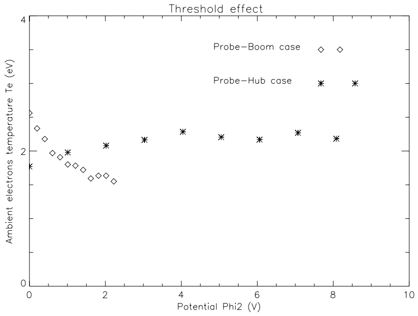

Figure 8. Estimate of the threshold for the ambient electron temperature above which the photo-electron cross current enhances the probe current. The expression of the cross current was computed by numerical integration.

5. Conclusion

A numerical scheme has been developed such has to compute the current between two interacting collectors in a magneto-plasma. The main results of *Hilgers* [1995] that were based on strong simplification by using mono-kinetik terms have been nevertheless confirmed by the more sophisticated numerical scheme. In particular, the existence of a temperature threshold that depends very little on the potential for observing the photo-electrons from the hub of the satellite has been confirmed. Thus the occurrence of a peak of the hub's photoelectrons is more likely to be related to the high ambient electron temperature existing inside the auroral arcs, rather than to the low ambient electron density. However, for the interaction between the probe and the boom the threshold depends both on the potential and on the temperature.

In conclusion, it is confirmed that at an altitude of a few thousand kilometers in the auroral zone (i.e., in a Viking-like experiment), the electron temperature can be a critical parameter determining the relative magnitude of the probe-hub interactions, whereas the electron density (via change in the satellite potential) has a stronger influence on the probe-boom interactions. This may explain why the peaks of photoelectrons are mainly observed inside the auroral acceleration structures and also why only one type of photoelectrons (the ones coming from the boom) can sometimes enhance the probe current while the other (the hub's photoelectrons) is not observed at all.

Besides the demonstration that $P(u)$ is mainly a function of the velocity, at least in the domain of parameters investigated, provide a semi-geometric formulation of current to elementary surfaces of a general form $P_i(u)S_i(u)$J that may be useful for several type of problems. In particular, this may be used to take into account the influence of the magnetic field in numerical charging analysis tools with a low cost of numerical computation.

6. References

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