SHEATH WAVES ON CONDUCTORS IN PLASMA AND THEIR IMPLICATIONS FOR LOW-EARTH-ORBIT SYSTEMS

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Abstract

The ion sheath, a region of low electron density that exists adjacent to any material surface immersed in a plasma, provides a channel for wave propagation when the surface is a conductor. For a sheath represented as a vacuum gap next to homogeneous, isotropic, cold plasma, sheath waves can propagate from zero frequency up to $1/\sqrt{2}$ times the plasma frequency. In anisotropic, cold plasmas with a magnetic field parallel to the surface, it is known that sheath waves can propagate in the direction of the magnetic field from zero frequency up to $1/\sqrt{2}$ times the upper-hybrid frequency. The theoretical basis for these properties and the supporting laboratory evidence is reviewed in this paper, with emphasis on computed and measured dispersion curves.

The first results of the relevant parts of the OEDIPUS rocket-borne tether experiment of January 1989 are presented. Early in the flight, the rocket separated at its midpoint into two halves connected by a thin wire referred to as the "tether" which was unreeled to a maximum length of almost one kilometre. In the experiment, the tether was maintained in a direction nearly parallel to the ambient magnetic field, and the rocket was launched into a region of auroral activity so that other researchers could use the tethered system to measure magnetic-field-aligned auroral electric fields. The experiment of primary interest in this paper was the transmission of a swept-frequency HF signal along the tether, and the main result is clear evidence of a sheath-wave passband from zero frequency to approximately $1/\sqrt{2}$ times the upper hybrid frequency. Within the passband, resonances at tether lengths that are multiples of a half-wavelength give the phase velocity of sheath waves along the tether. Other pass and stop bands appear to be associated with electron cyclotron harmonic frequencies. It is concluded that sheath waves will have to be taken into account when estimating the coupling of electromagnetic interference between any two points on any large space structure in low orbit.

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Introduction

When a plasma is in contact with a metallic surface (or almost any other material surface), it is well known that there exists near the surface a region known as the sheath in which the electron density is much lower than in the surrounding plasma. Ions predominate in the sheath but, because they are so much more massive than the electrons, their inertia makes them nearly immobile at the frequencies to be considered here. If one views the sheath region adjacent to a good conductor as a vacuum gap, and if one views the plasma rather loosly as a poor conductor at low frequencies, the existence of electromagnetic waves in the sheath region between these two conductors seems entirely plausible. Indeed, sheath waves do exist and their properties have been studied for over twenty-five years.

Sheath waves can be understood by considering first a planar interface between a vacuum and a homogeneous, isotropic, cold plasma. It is straightforward to show that electromagnetic surface waves can exist, decaying exponentially to either side of the interface, and propagating at a velocity slower than that of a uniform plane wave in a vacuum. As the frequency is raised, the decay rates increase and the wave becomes more tightly bound to the interface, until cutoff is reached at $1/\sqrt{2}$ times the plasma frequency. Now, if a perfectly conducting surface is introduced in the vacuum region such that a vacuum gap separates the perfect conductor from the plasma, it is also straightforward to show that waves can propagate from zero frequency up to the same cutoff frequency just mentioned. At low frequencies, the fields are concentrated in the vacuum gap region, and the wave propagation is almost dispersionless, exhibiting a phase velocity lower than that of light in a vacuum. As the frequency is raised, the phase velocity decreases, and the fields become more tightly bound to the vacuum-plasma interface, resembling more and more the fields that exist in the absence of the perfect conductor, until the cutoff frequency is reached. The tight binding of the fields to the planar vacuum-plasma interface at frequencies somewhat below cutoff suggests that a model with a more realistic, gradual transition from sheath to plasma could have a strong effect on predictions of wave propagation near cutoff and on the predicted cutoff frequency itself.

Some historical background is in order. *Seshadri* [1965] calculated the dispersion of sheath waves on a cylindrical conductor, using a vacuum-gap sheath and warm plasma theory. *Miller* [1968] did a similar study for an infinite dipole antenna, including a steady magnetic field. Experimental studies were included in the papers by *Ishizone et al.* [1969, 1970a, 1970b], *Lassudrie-Duchesne et al.* [1973], *Meyer et al.* [1974], *Marec* [1970, 1974] and *Marec and Mourier* [1970, 1972]. *Mushiake* [1964] derived the dispersion relation for surface waves along a thin, unsheathed wire parallel to a steady magnetic field, and *Adachi* [1977] proposed a transmission-line theoretical model to calculate the impedance of a sheathless dipole antenna with an arbitrary orientation relative to the magnetic field. *Laurin* [1986] and *Laurin et al.* [1989] deduced sheath-wave dispersion relations from measured antenna input impedance for the case of a cylindrical monopole antenna parallel to the magnetic field, and made a qualitative comparison with calculations for a planar geometry involving a vacuum-gap sheath and a cold magnetoplasma.

Isotropic Plasma

A few key results from the work of *Morin* [1987] will be summarized here, on the propagation of sheath waves along a cylinder in a warm, inhomogeneous, isotropic plasma. Figure 1 shows the cylinder together with qualitative graphs of electron density profiles. The continuous density profile closely represents physical reality and ranges from N_s at the probe surface to the ambient electron density N_0 at a great distance. In a computational model, the continuous profile could be approximated by a sequence of closely spaced steps, with the validity condition that the computed results would smoothly approach the results for the continuous profile as the number of density steps was increased. The minimal stepwise approximation is the single-step case shown in Figure 1, in which the sheath-region density is N_s, and in which the sheath radius r_1 has to be established by some means such as comparing a single-step calculation with a calculation using the continuous density profile (this usually fixes r_1 at or very close to the point where the sheath electron density is half the ambient density). In many situations, especially when the cylinder has a moderate to high negative voltage bias with respect to a different electrode elsewhere in the plasma, the sheath density is low enough to be approximated as zero, giving the "vacuum-gap" sheath representation.

A typical continuous sheath profile (for high negative bias) taken from *Laframboise* [1966] is shown in Figure 2 along with a multi-step approximation to it. Morin's calculations of the sheath-wave complex wavenumber are shown in Figure 3 for a single-step approximation to the continuous density profile, and in Figure 4 for the multi-step approximation. The real parts in the two figures are nearly identical at frequencies ranging from zero up to about 1/3 of the plasma frequency, indicating nearly dispersionless propagation at about 0.3 times the velocity of light. The cutoff frequency can be associated with the peak in the attenuation curve, so the cutoff frequency is about 0.83 ω_p in the single-step case and 0.55 ω_p in the multi-step case. These two values derived from warm-plasma theory straddle the expected cutoff frequency of 0.71 ω_p deduced from the elementary cold-plasma, vacuum-gap theory described earlier, and the low-frequency phase velocities are essentially the same as the elementary cold-plasma vacuum-gap values.

An experimental program was carried out by Morin using the arrangement shown in Figure 5. The DC bias circuit puts a small positive bias on the outer conductor of the coaxial cable, relative to the discharge-tube anode. This collapses the sheath around the cable outer conductor, thereby attenuating sheath waves along it and preventing coupling into the laboratory environment. The complex wavenumber was measured by using multiple lengths of the extended inner conductor of the coaxial cable as a monopole antenna, achieved without affecting the plasma, by pulling the inner conductor out and cutting off the excess wire before re-attaching an RF connector, for each monopole length. The experimental results are shown in Figure 6, having been corrected for unwanted system reflections. The experimental conditions were: wire length = 20.05 to 7.85 cm; wire radius = 0.45 mm; cable outer diameter = 3.6 mm; cable bias = 4.0 V; helium gas at a pressure of 80 m Torr; electron temperature = 0.19 eV; plasma frequency = 1030 MHz; wire bias = -2.0 V = floating potential. The corresponding theoretical calculations were done using the multi-step warm-plasma profile of Figure 7, producing the computed results of Figure 8 which are very close to the experimental results of Figure 6.

It will be noted in the profile of Figure 7 that there is a large density jump to the ambient density, a jump that does not approximate the quite gradual continuous density profile. It was established that this density jump does not affect the wavenumber calculations at low frequencies (although it would have a large effect at frequencies approaching cutoff). The reason for this is the concentration of low-frequency wave fields in the region between the metallic surface and the point in the sheath where the electron density is half the ambient density, which makes exact modelling of the outer region of the sheath less important. Therefore it is concluded that the warm-plasma multi-step density profile model is valid at low frequencies for isotropic plasmas.

Anisotropic Plasma

The propagation of sheath waves in a direction parallel to the magnetic field has been investigated by *Laurin et al.* [1989]. Typical cold-plasma theoretical results for a planar geometry and a vacuum-gap sheath are shown in Figure 9. The main result is that, relative to the isotropic case, the cutoff frequency has risen to $\omega_{uh} / \sqrt{2}$ where ω_{uh} is the upper-hybrid frequency. The low-frequency sheath waves are nearly dispersionless as before, with a phase velocity about 1/4 the velocity of light in a vacuum. Experiments were done on a wire monopole antenna in a magnetized laboratory plasma, and qualitative agreement with the planar theory was established. As one might expect, the phase velocities for the wire monopole were higher than for the planar geometry, the wire sheath-wave velocities being of the order of 1/2 the velocity of light in vacuum. An interesting observation was made in both theory and experiment, that the wavenumber tended to be independent of the ambient density at frequencies in the vicinity of the cyclotron frequency.

The "OEDIPUS A" Rocket Experiment

The OEDIPUS experiment's primary mission was to measure magnetic-field-aligned electric fields in the ionosphere under auroral conditions. The procedure was to create a double probe by having the rocket separate into two parts connected by a thin, insulated wire (or "tether"), and to let the wire unreel to a maximum length of about one km, all the while keeping the wire nearly parallel to the earth's magnetic field.

This configuration was ideal for a study of sheath waves propagating parallel to the magnetic field, so a stepped-frequency pulsed transmitter and a synchronized receiver were added to the nose and tail sections of the rocket, as shown in Figure 10. The transmitter output was fixed at 50 volts rms. Between the transmitter and the spool, a 2000 ohm resistor was inserted to reduce tether input current variations with frequency. The receiver input impedance was 100 ohms. The receiver samples the intermediate-frequency signal and post-flight processing of the sampled data involves squaring and averaging over a time window around the received pulse. The tether/spool subsystem specifications are given in Table 1.

The OEDIPUS A rocket was launched from Andoya, Norway on January 30, 1989. The tether unreeled as planned, reaching a maximum length of just under 1 km about half-way through the flight, as indicated in Figure 11 which also shows a maximum altitude of just over 500 km, so that for most of the flight the rocket was above the F-region maximum in electron density.

The main result of the flight, from preliminary processing of the data, is the gray-scale plot of Figure 12 in which there are 8 gray levels in equal increments on a log scale. The darker the gray shading, the higher the signal level at the receiver in the tail section. The lines superposed on the figure are the cyclotron frequency f_c with its 2nd, 3rd and 4th harmonics, as well as the plasma frequency f_p deduced from Langmuir probe measurements, the upper hybrid frequency f_u deduced from f_p and f_c, and the estimated sheath cutoff frequency $f_s = f_u / \sqrt{2}$. The strongest passband extends from zero frequency up to about f_s as expected for sheath wave propagation. Other passbands and stopbands appear to be bordered by the cyclotron harmonic frequencies, reminiscent of cyclotron-harmonic waves (Bernstein waves) that propagate at right angles to the magnetic field.

Within the lower passband, interference fringes are visible and are shown in Figure 13 after enhancement by gray-scale adjustment. It was speculated that these might be sheath-wave resonances, so lines are shown on Figure 13 for the conditions that the tether length is an integral multiple of a half-wavelength, assuming a phase velocity 2/3 the velocity of light in a vacuum (i.e. the sheath-wave refractive index is 1.5). The excellent fit suggests strongly that the sheath-wave resonance postulate is correct. Moreover, the easy visibility of these resonances indicates that the sheath waves are not greatly attenuated.

Figure 14 shows frequency sweeps at four times during the flight. The passbands, stopbands and resonances are clearly visible, with the resonance nulls being deeper at the shorter tether lengths. The first stopband is remarkably deep, in most cases over 60 dB below the lowest-frequency passband.

Moment-Method Calculations

A thin-wire method-of-moments computer program written by *Richmond* [1974] and improved by *Tilston* [1989] is applicable to wires in isotropic cold plasma with a thin vacuum-gap sheath around the wire. A number of multi-frequency calculations of the input impedance of dipoles in plasma served to identify resonances and deduce phase velocities, using the actual tether wire size and a 2.5 cm estimate of the actual sheath thickness. The purpose was to get an estimate of the phase velocity of sheath waves for realistic parameters but ignoring the plasma anisotropy. Figure 15 shows two curves, one obtained by equating the plasma frequency in the moment-method calculation with the ionospheric plasma frequency, and the other obtained by equating the plasma frequency in the moment-method calculation with the ionospheric upper-hybrid frequency. The sheath-wave wavenumbers deduced from the various resonances in Figure 13 are also shown, and the fit is especially good when the upper-hybrid frequency is employed in the moment-method calculation. The values of the wavenumber (refractive index) are clustered around the 1.5 value used in Figure 13 for the resonance lines. Besides lending some credibility to the sheath-wave interpretation of the data, this moment-method analysis also suggests that isotropic-plasma calculations may have a certain degree of utility in estimating sheath-wave behaviour, in the absence of a rigorous anisotropic-plasma theory.

Relevance to EMI Coupling

On large structures in the ionosphere (such as the Space Station) the possibility exists that electromagnetic interference emitted at one location will be coupled to other locations via sheath waves. The evidence already presented indicates a broad sheath-wave passband up to 1.5 to 2.0 MHz in which electromagnetic waves can propagate with little dispersion and little attenuation, so it is clear that transient signals could propagate easily as sheath waves provided that their spectra are concentrated below 2 MHz.

Estimates of electromagnetic coupling and electromagnetic compatibility standards are generally based on the assumption of a vacuum (air) environment. A comparison between vacuum and plasma media can be made using the moment-method program already referred to. The configuration representing the tether is shown in Figure 16 along with the computed results for vacuum and plasma, and for two different tether lengths. The plasma frequencies selected correspond to the two tether lengths. The same 2.5 cm sheath thickness was assumed. The crucial result is that, at frequencies below 1 MHz, coupling in the plasma medium is 20 dB to 60 dB greater than it is when the surrounding medium is a vacuum. This suggests that estimates of EMI coupling between points on large space structures could be too low by a large margin if they do not take into account the plasma environment and the existence of an ion sheath. Moreover, sheath-wave resonances could increase coupling if the structure is long enough.

Conclusions

Reported in this paper is what is believed to be the first measurement of sheath wave propagation along a wire in the ionosphere. The low-frequency passband and the first stopband are in the frequency range predicted theoretically, and the phase velocity deduced from resonance frequencies has a value essentially as expected from various theoretical considerations. The existence of readily identifiable resonances indicates that sheath waves propagate with little attenuation. The strength of all the phenomena measured suggests that sheath waves are easy to excite. Moreover, sheath-wave phenomena are more complex than expected, as evidenced by the observation of passbands and stopbands bordered by the electron cyclotron harmonic frequencies.

It is clear that sheath waves must be taken into account when predicting or interpreting the properties of antennas in the ionospheric plasma, certainly at all frequencies below 5 or 6 MHz. In anisotropic plasma, sheath-wave propagation has been identified even in frequency ranges where the medium can propagate uniform plane waves, which is in contrast to the isotropic plasma case where sheath waves can propagate only at frequencies appreciably below the electron plasma frequency.

The properties of sheath waves are such that they could readily carry electromagnetic interference between any two sites on a large structure in the ionosphere, and the coupled power level could be orders of magnitude higher than in free space. This means that estimates of electromagnetic interference levels could be grossly incorrect if they do not take the plasma medium into account. Therefore EMI/EMC standards need to be reviewed to see if they are applicable to large structures in the ionosphere. Useful estimates of sheath wave interference effects can be deduced from moment-method computational techniques valid for a cold, isotropic plasma.

EMI/EMC standards for space systems relate not only to theoretical estimates of interference levels in space, but also to ground-based compliance testing. The analogy between sheath-wave propagation and coaxial cable propagation suggests a configuration for ground-based testing. It would involve wrapping space devices with an appropriately modified wire mesh spaced a distance of one sheath width (a few cm) from the device using some material such as polystyrene foam for support. There remains the necessity to establish the details of the wire mesh modification required to ensure the validity of this procedure, that is, there remains the necessity to establish an equivalence between the wire-mesh sheath edge and the physical sheath-plasma transition region.

Acknowledgment

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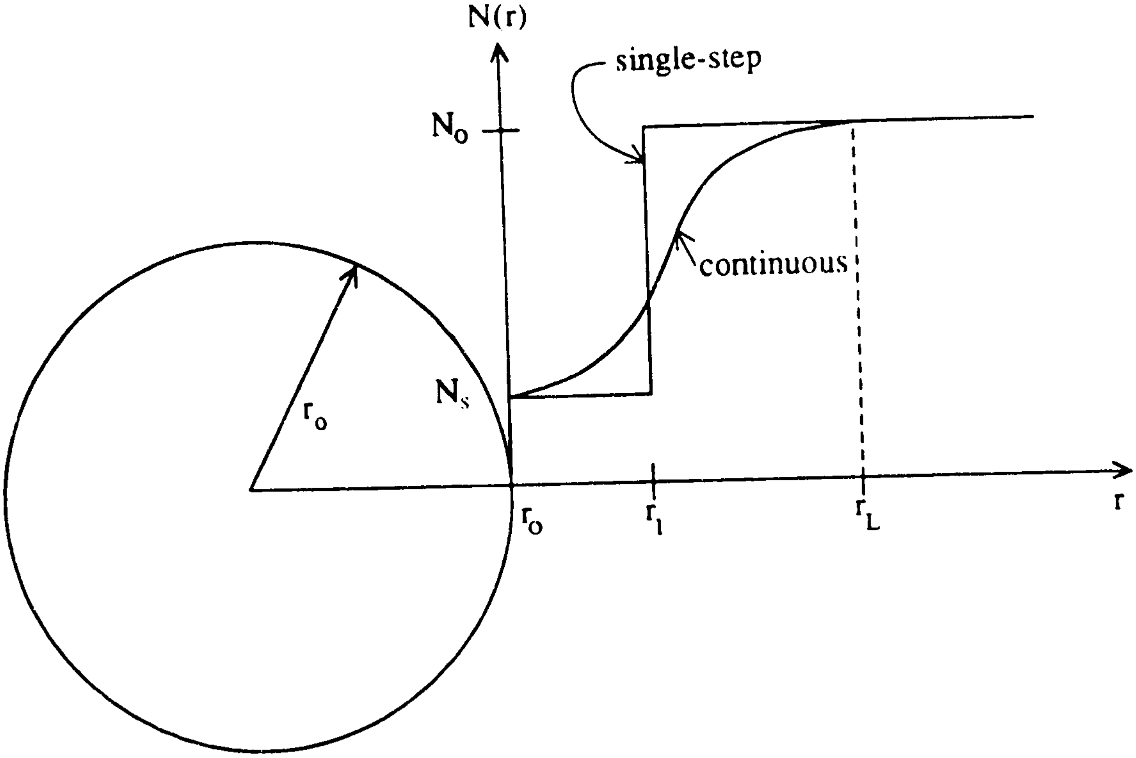


Figure 1. Cross-section of cylindrical conductor in plasma with sheath profiles of electron density.

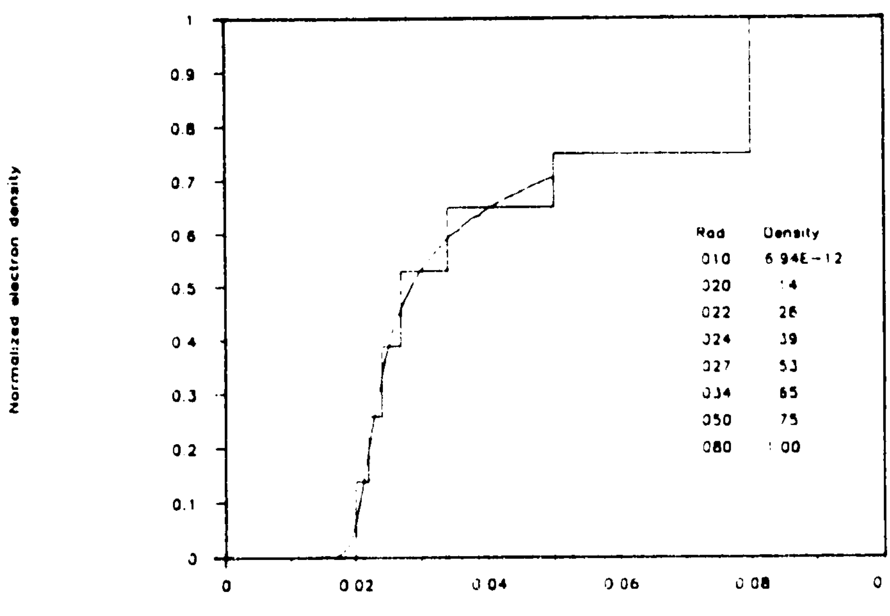


Figure 2. Electron density profile for -25V bias as calculated by *Laframboise* [1966].

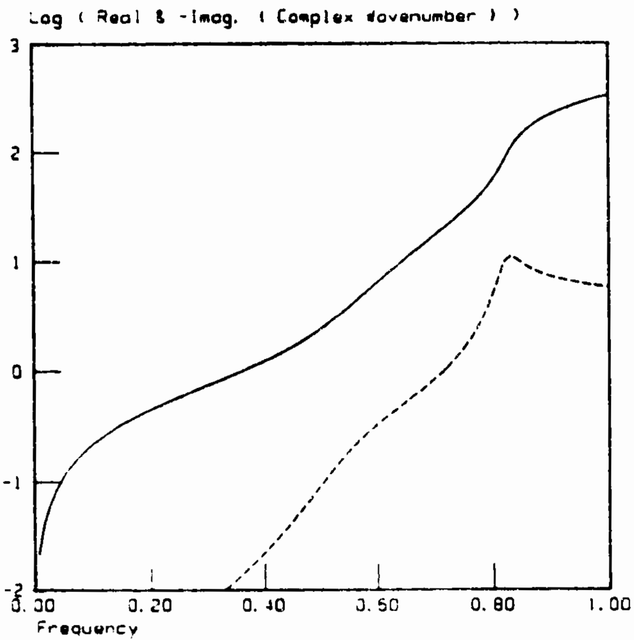


Figure 3. Complex wavenumber for -25V bias (solid line is real part, dashed line is imaginary part representing attenuation) for single-step density profile. Norm. collision freq. = .01 = ν / ω_p, norm. thermal vel. = .001 = V_{th} / c, norm. probe radius = .01 = $r_o \omega_p / c$, norm. sheath radius = .028 = $r_1 \omega_p / c$, sheath electron density = 6.94×10^{-12} cm^{-3}.

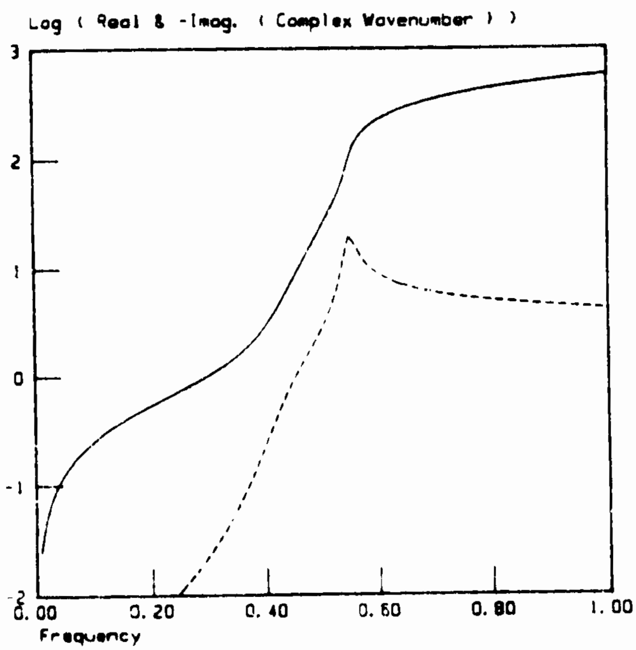


Figure 4. Complex wavenumber for -25V bias for multi-step density profile as in Figure 2. Parameters same as in Figure 3.

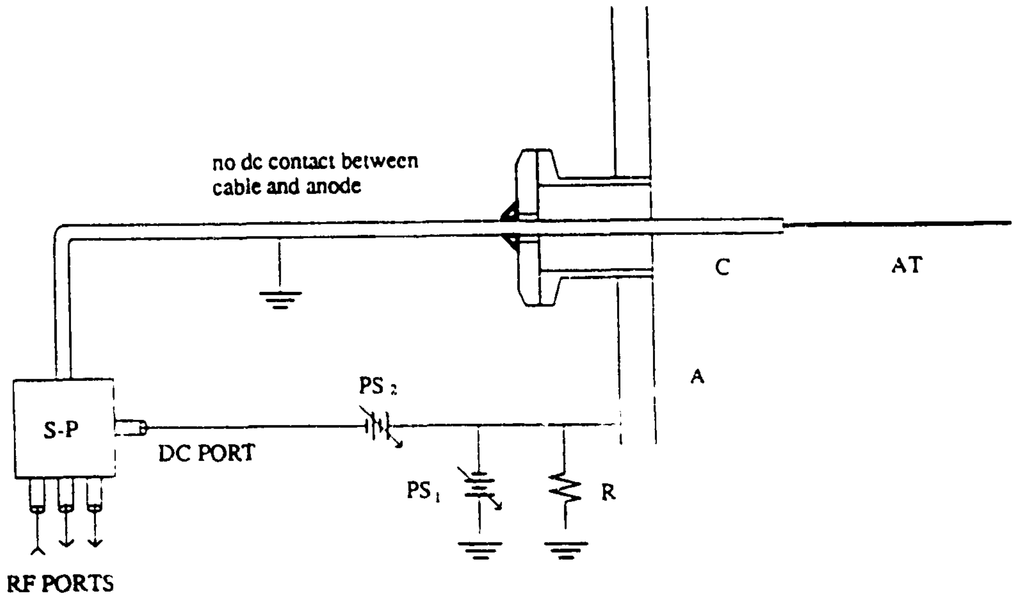


Figure 5. Antenna in laboratory plasma, showing cable biasing circuit. A denotes anode, AT antenna, C cable, R = 1500 Ω, PS_1 cable bias, PS_2 antenna bias, S-P is S-parameter test set.

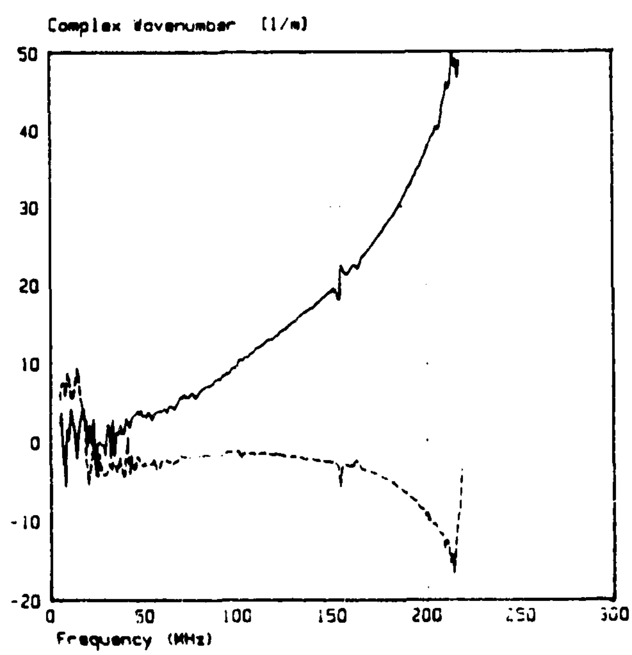


Figure 6. Experimental complex wavenumber as corrected for stray reflections (crosses show uncorrected real part obtained from specific resonances).

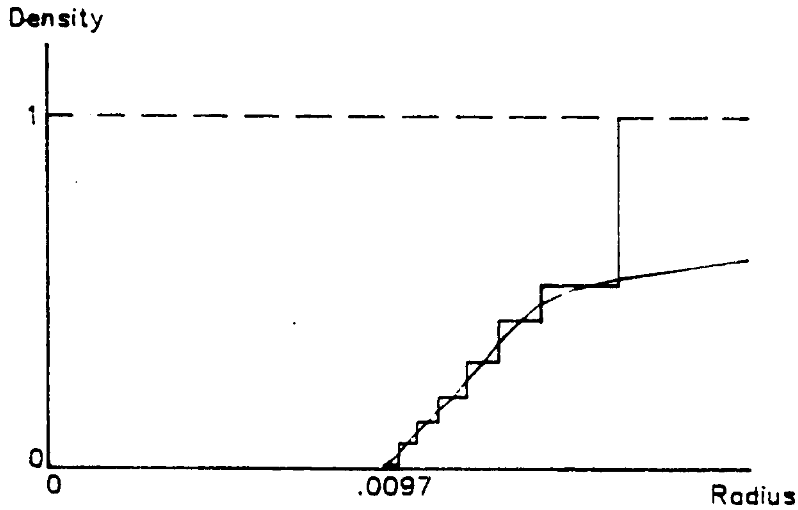


Figure 7. Theoretical electron density profile used to represent laboratory experimental conditions.

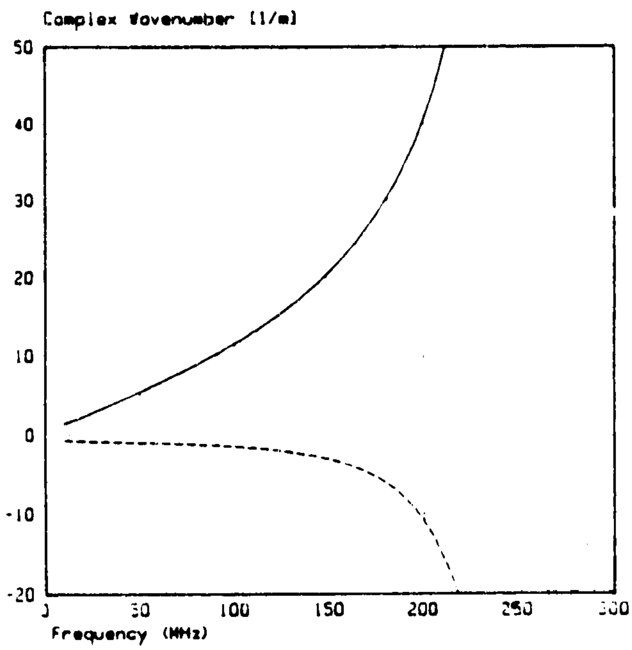


Figure 8. Calculated waenumber for profile of Figure 7.

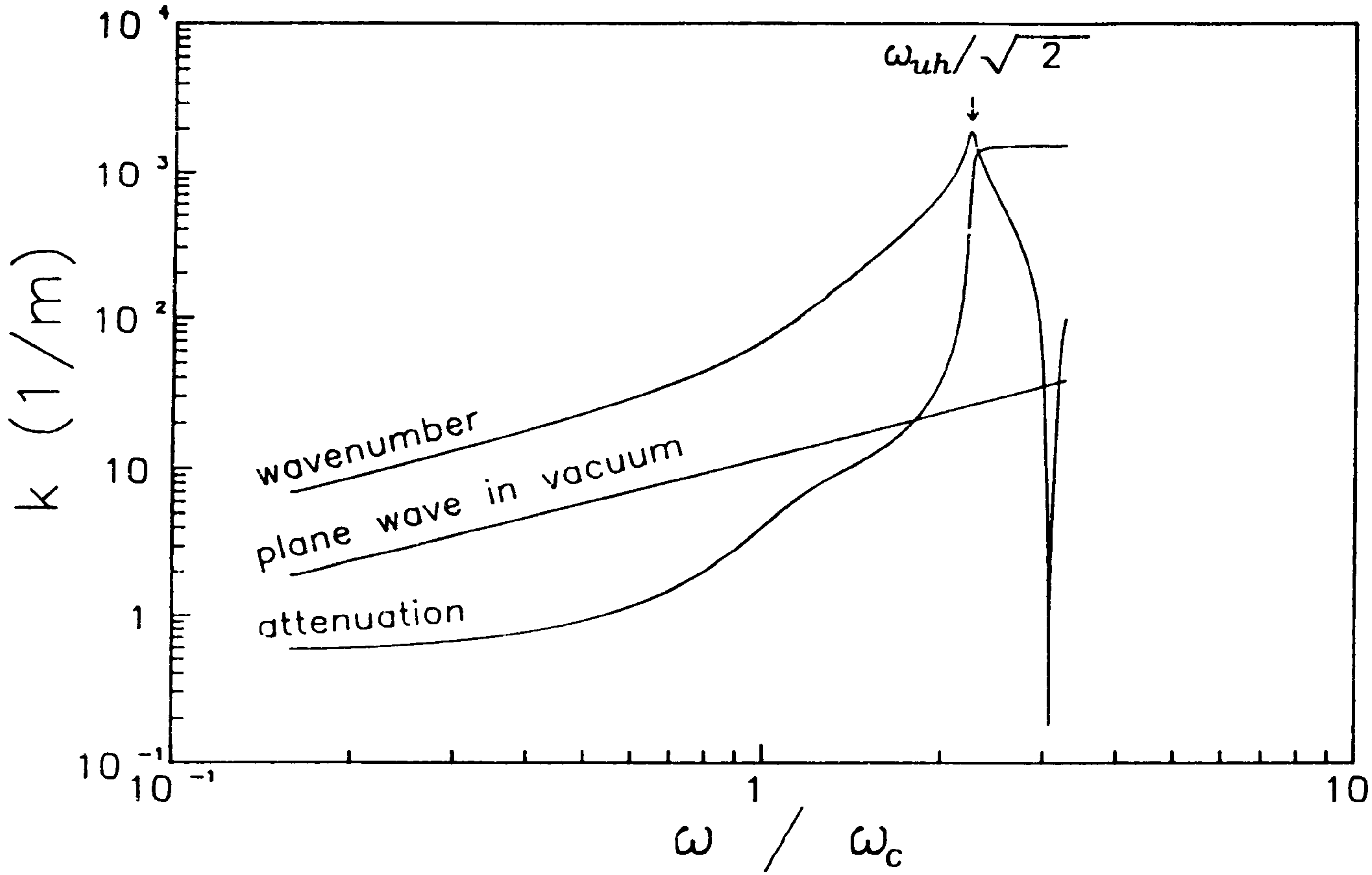


Figure 9. Typical theoretical sheath-wave dispersion curves for planar geometry and magnetic field in direction of propagation. $\omega_c = 3.52 \times 10^9$ rad/s, $\omega_p = 3\omega_c$, $\nu = \omega_c / 28$, sheath thickness = 0.1 cm.

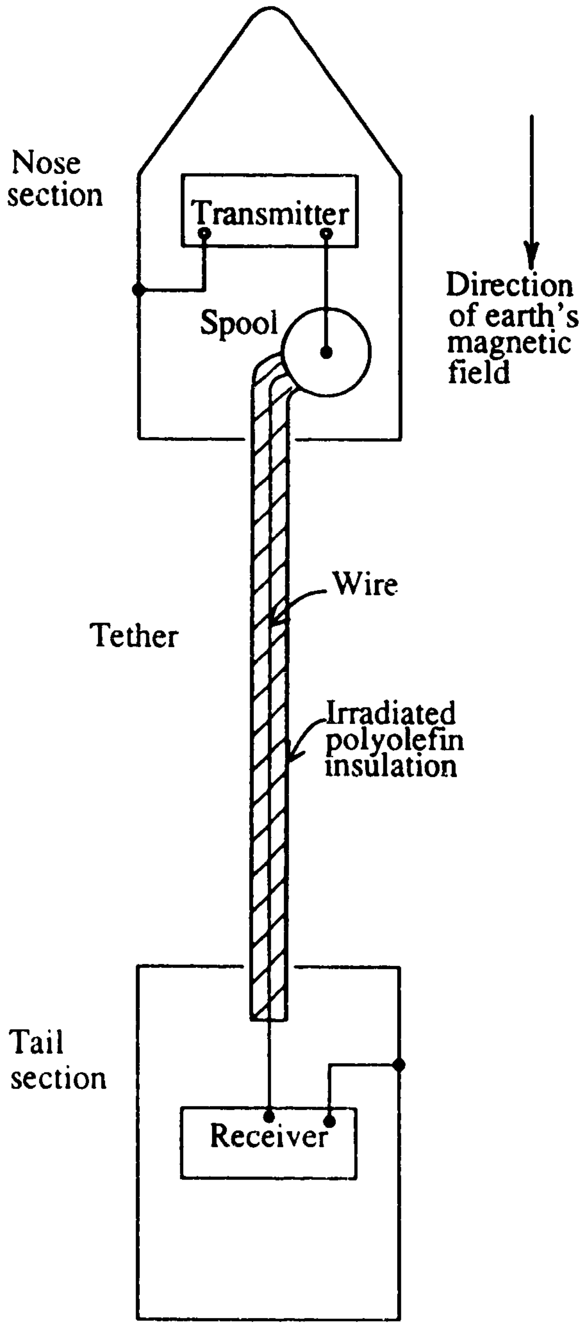


Figure 10. Diagram of the OEDIPUS A rocket experiment configuration used for the study of sheath waves.

Table 1. Tether Spool Subsystem

Tether wire length	1300 meters
Tether wire	No. 24 AWG, 19 strands of No. 36 copper, diameter 0.020"
Tether wire coating	irradiated polyolefin, diameter 0.057", $\varepsilon_r = 2.32$
Wire resistance	< 100 ohms over 1300 meters
Spool - to - chassis resistance	> 3×10^{14} ohms
Contact	slip ring
Braking	constant - torque of 6.5 oz - in

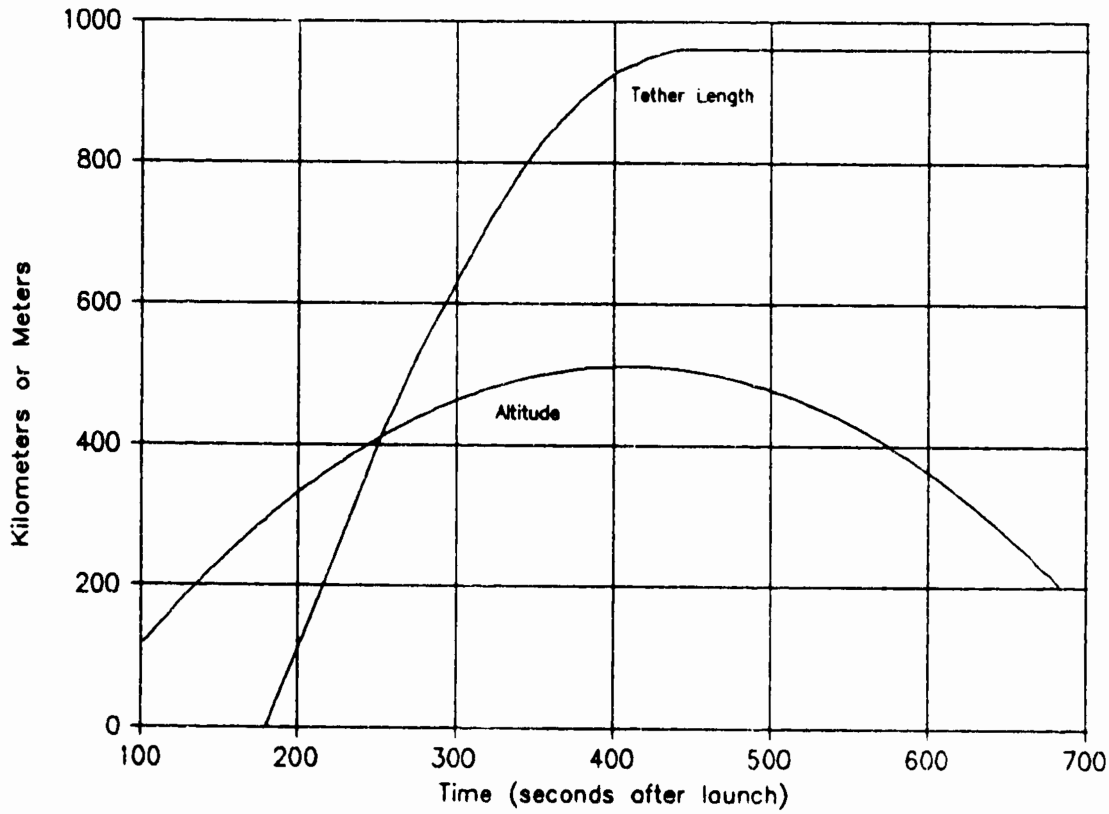


Figure 11. Tether length and altitude as functions of elapsed time.

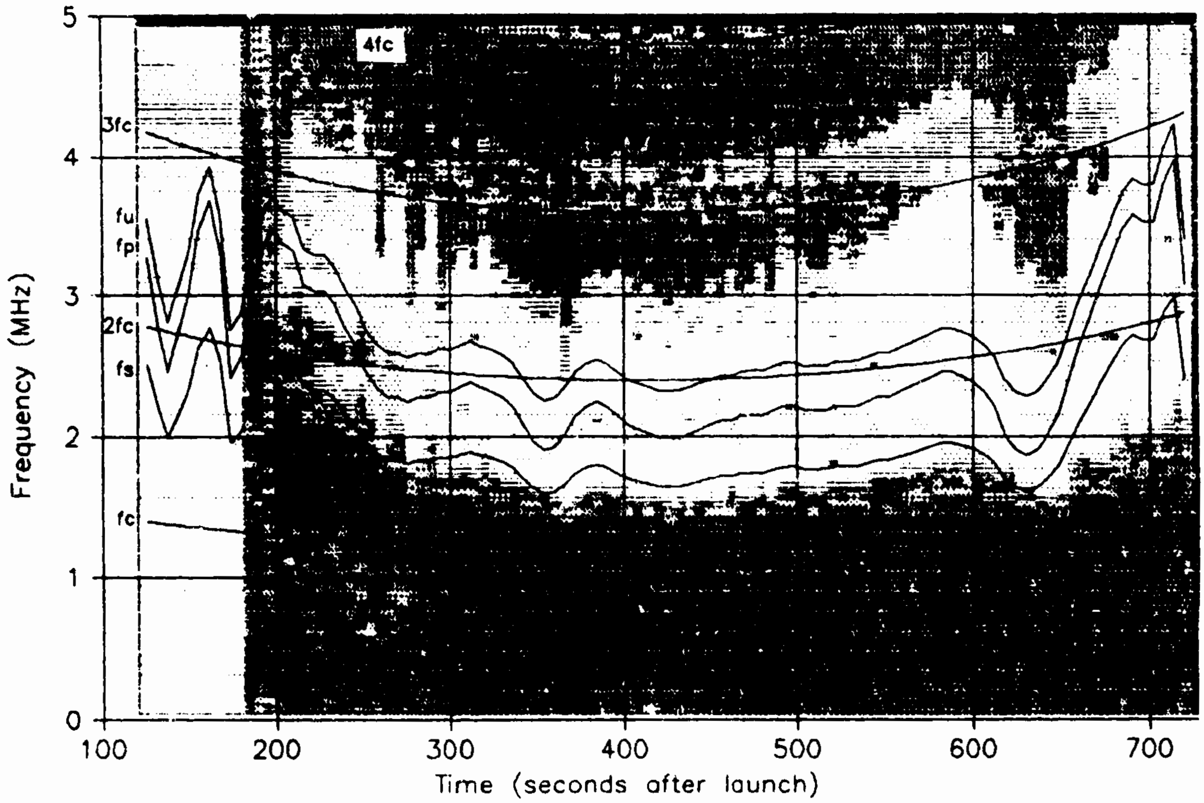


Figure 12. Received signal strength (darker gray scale means higher signal level) showing the cyclotron frequency f_c and its harmonics, the plasma frquency f_p, the upper-hybrid frequency f_u, and the nominal sheath-wave cutoff frequency $f_s = f_u / \sqrt{2}$.

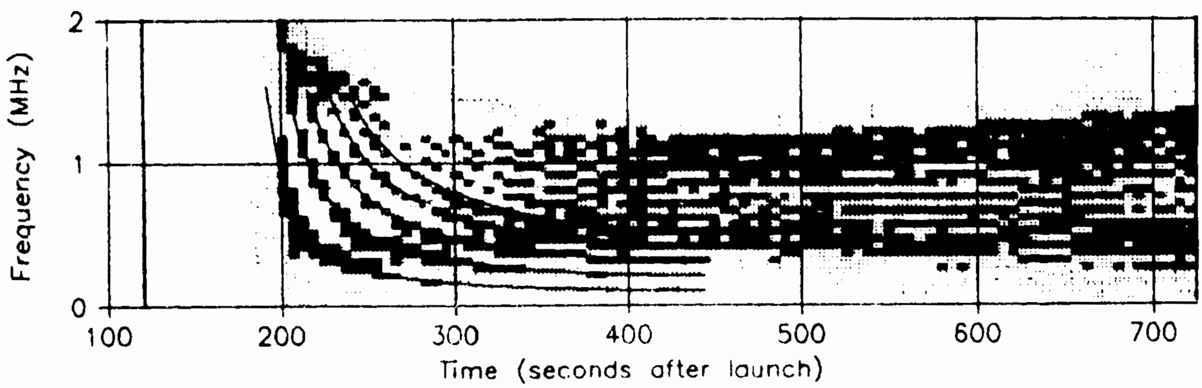
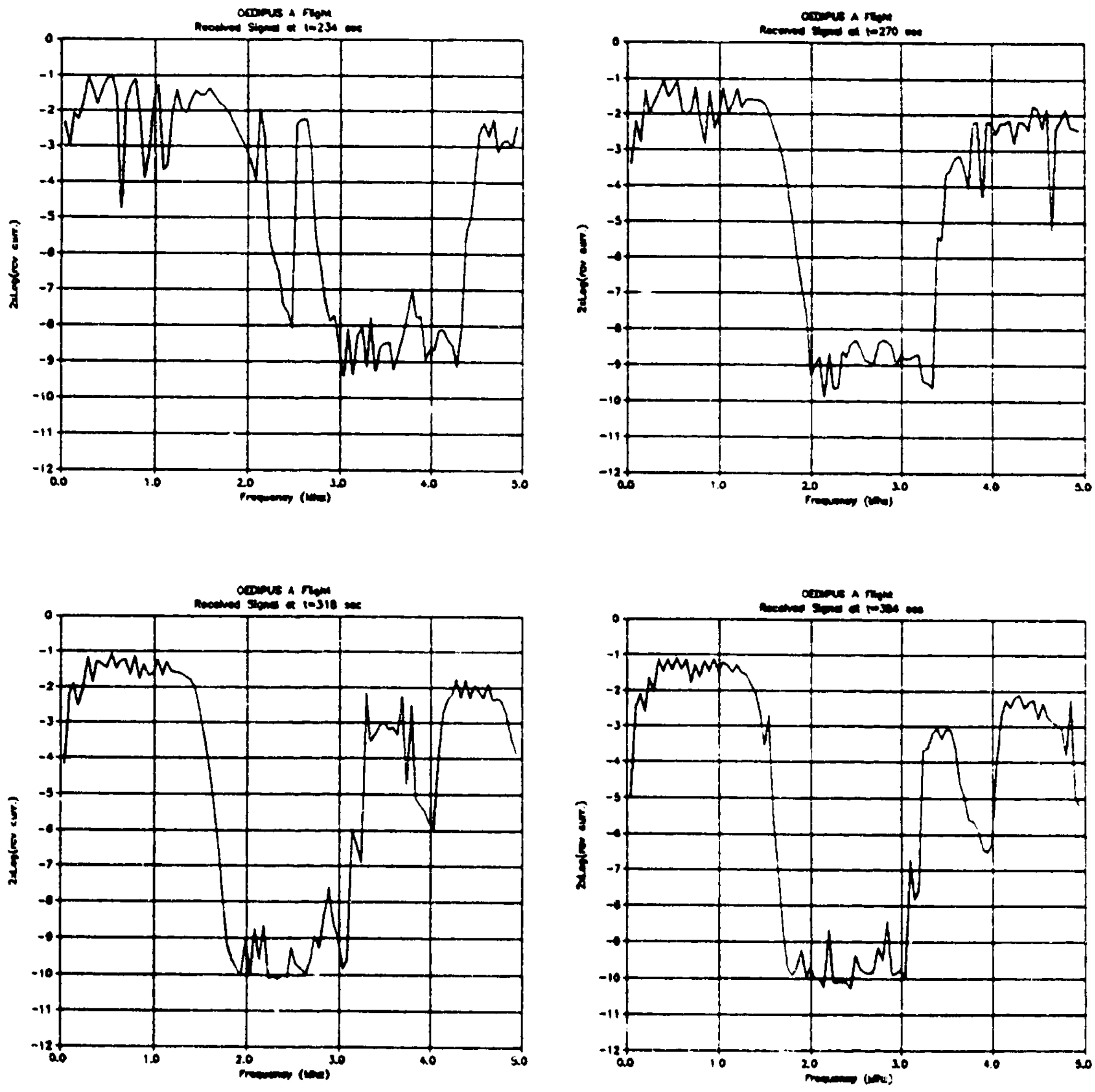


Figure 13. Received signal strength with gray-scale adjusted to show tether-length resonances. Lines show where tether is a multiple of a half wavelength long, assuming a wavenumber of 1.5.

Figure 14. Frequency sweeps of received signal level at elapsed times 234 sec., 270 sec., 318 sec., and 384 sec.



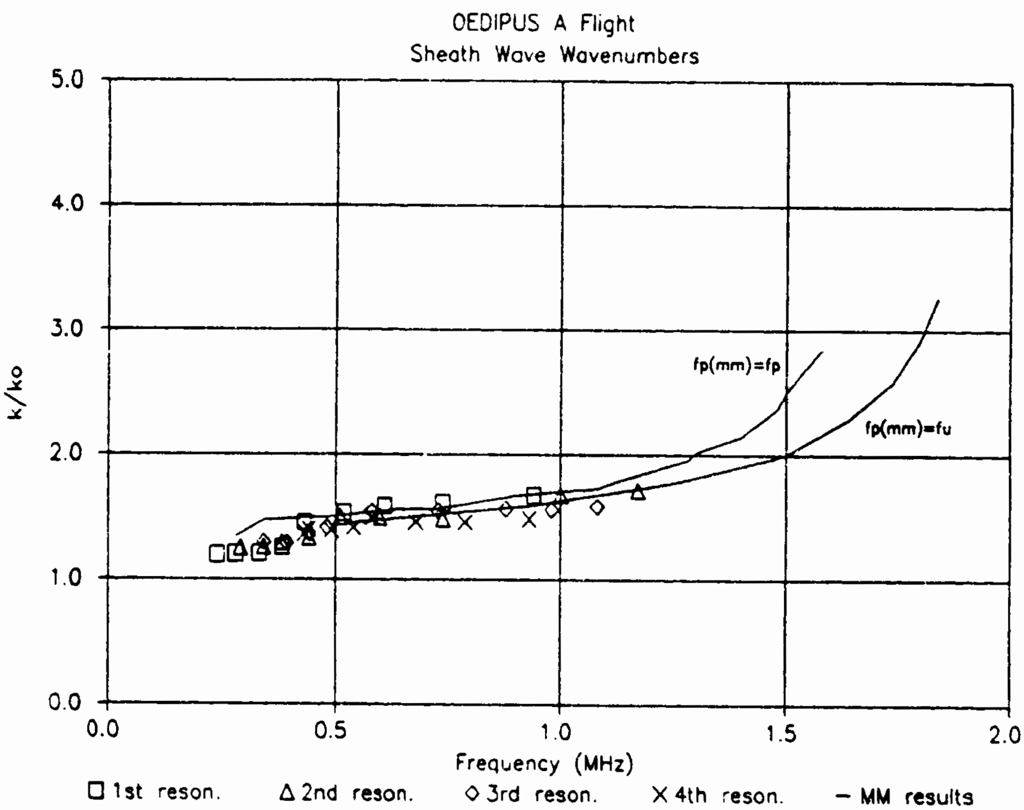


Figure 15. Wavenumbers derived from various resonances, together with moment-method calculations of wavenumbers using two different ways to specify the ambient plasma frequency.

Figure 16. Moment-method calculation of coupling from one end of the tether to the other, for both free space and plasma environments and for two different tether lengths. The plasma densities correspond to the tether lengths chosen.