Three-Dimensional Computer Models of the Currents Collected by Active Spacecraft In Low Earth Orbit

I. Katz, M. J. Mandell, G. A. Jongeward, J. R. Lilley and V. A. Davis

S-CUBED Division of Maxwell Laboratories, Inc., La Jolla, CA

David L. Cooke

Geophysics Laboratory/PHK, Hanscom Air Force Base, MA

There has been much progress in determining the physical mechanisms that control the flow of charged particles from the ionosphere to spacecraft at potentials large compared with the ambient plasma temperature. At altitudes greater than a few hundred kilometers, space charge dominates ion collection, while both space charge and magnetic fields limit electron collection. At lower altitudes, ion collection remains space-charge limited, but electron collection is dramatically enhanced by ionization within the sheath. The NASCAP/LEO and POLAR codes solve Poisson's equation, in three dimensions, for the potential in the vicinity of arbitrary geometry spacecraft. The plasma currents collected by spacecraft are calculated by tracking representative particle orbits through the sheath. The validity of the code's underlying algorithms are discussed and comparisons with flight experiments are shown. A comparison of CHARGE-2 calculated and measured currents shows that both the 3-dimensional geometry and the self-consistent space charge are important to obtain agreement with the flight data. SPEAR I calculations show that the interaction between ion and electron collecting sheaths breaks the symmetry and permits more electrons to be collected than the spherical probe theory would suggest.

INTRODUCTION

During the past decade, there has been much discussion about how spacecraft collect current from the ionosphere (Winckler, 1980). Even though the basic equations describing the plasma surrounding a high-voltage spacecraft are well known, there has been debate on what set of algorithms is both sufficient to describe the plasma interactions, and yet practical enough to provide answers in a reasonable time on available computers (LaFramboise, 1982). Recently, the NASCAP/LEO (NASA Charging Analyzer Program for Low Earth Orbit) and POLAR (Potential Of Large spacecraft in the Auroral Region) codes have successfully been used to model the current collecting sheaths of the SPEAR I and CHARGE-2 sounding rockets. The algorithms employed by these two computer codes provide insight into the mechanisms that control current collection from the ionosphere. The upcoming electrodynamic Tethered Satellite System (TSS-1) will have sufficient instrumentation to further test the adequacy of the physics in these computer codes.

THEORY

The basic approach employed is to consider a spacecraft as a large, asymmetrical, high-voltage probe immersed in a magnetoplasma. There has been extensive research into the current characteristics of probes in plasmas (Langmuir and Blodgett, 1924; Mott-Smith and Langmuir, 1926; Beard and Johnson, 1961; Chen, 1965; Laframboise and Rubinstein, 1976; Rubinstein and Laframboise, 1978, 1982, 1983; Parker and Murphy, 1967). Most of the published work has been for symmetric probes. The emphasis in this discussion is on the extension of the basic theories into algorithms that account for asymmetries and the earth's magnetic fields. The approach used is to examine the conservation laws that limit current collection, and to identify the ones that most limit the current. The algorithms in the computer codes satisfy the most severely limiting conservation laws.

The ionosphere is a cool dense plasma. For the sounding rockets, typical ionospheric plasma parameters are

$$n_e = n_i = 10^{11}\ \mathrm{m}^{-3}$$
$$\theta_e = \theta_i = 0.1\ \mathrm{eV}.$$
$$B = 0.4\ \mathrm{Gauss} \tag{1}$$

The time and distance scales associated with this plasma are

$$\omega_{pe} = 2 \times 10^7\ \mathrm{sec}^{-1}$$
$$\omega_{ce} = 7 \times 10^6\ \mathrm{sec}^{-1}$$
$$\lambda_D = 0.007\ \mathrm{m}$$
$$\lambda_{ce} = 0.02\ \mathrm{m}$$
$$\lambda_{ci} = 3\ \mathrm{m}. \tag{2}$$

Typical active spacecraft experiments have dimensions of meters, potentials of hundreds of volts or more, and durations as long as seconds. For such spacecraft, the disparity between the time and distance scales of the spacecraft and those of the plasma is so great that direct simulation is not practical. Direct simulation, such as the use of Particle In Cell (PIC) codes, is appropriate when all the dimensions are comparable. The wide range of time and distance scales in the problem considered here allows approximations to be made that make the problem easier to solve than if the range were smaller.

The equilibrium state of the plasma surrounding a spacecraft can be described by Poisson's equation and the collisionless Vlasov equation,

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0} \tag{3}$$

$$\rho(\mathbf{x}) = e\left(\iiint f_i(\mathbf{x},\mathbf{v})\,d\mathbf{v} - \iiint f_e(\mathbf{x},\mathbf{v})\,d\mathbf{v}\right), \tag{4}$$

where ϕ is the potential and f_i, f_e are the ion and electron distribution functions, respectively. The potential is measured with respect to the unperturbed plasma at great distances. The discussion below first examines restrictions on the particles that can be collected by the spacecraft neglecting the charge density in equation (3). Then, the additional restrictions on the current due to finite space charge are considered.

If the range of the potential were infinite, the maximum impact parameter of particles collected would be limited by angular momentum. For a sphere of radius a and potential ϕ, plasma electrons with velocity v_{th} could be collected if their impact parameter, b, was less than the limiting value.

$$b \leq a\,(v_f/v_{th})$$
$$b \leq a\,(1 + \phi/\theta_e)^{1/2} \tag{5}$$

The collected current, which depends on b^2, increases linearly with potential. This type of collection is seen in hot, dilute plasmas, such as the magnetosphere, but is rarely observed in the ionosphere.

For electrons in the ionosphere, even if the range of the potential were infinite, the magnetic field introduces a canonical angular momentum that severely restricts the range of impact parameters which can be collected (Parker and Murphy, 1967),

$$p_\theta = mr^2\left(\frac{d\theta}{dt} + \frac{\omega_{ce}}{2}\right)$$

$$b \leq a\sqrt{1 + \left(\frac{8e\phi}{m\omega_{ce}^2 a^2}\right)^{1/2}} \tag{6}$$

Conservation of canonical angular momentum is typically the most limiting condition in the collection of electrons from the ionosphere. Early electron beam experiments aboard rockets reported currents much larger than implied by the Parker-Murphy theory, and others speculated that plasma turbulence scattered electrons across magnetic field lines (Linson, 1969). However, recent data at altitudes above 250 km show clear evidence of magnetic limiting. These results imply that the earlier results were due to ionization of the background neutral gas.

The angular momentum limits described above are predicated on an infinite range of the attracting potential. This condition is clearly violated in the ionosphere for spacecraft at high potentials. A space charge sheath forms around a high potential spacecraft. This sheath shields the bulk of the plasma from the potential. Since electric fields are very small in the surrounding plasma, the sheath satisfies the condition that the sheath space charge balances the spacecraft surface charge,

$$\iiint_{\text{sheath}} \rho\, dx + \oint_{\text{spacecraft}} \sigma\, dS = 0. \tag{7}$$

The space charge of the attracted electrons or ions would shield a ±8000 volt potential on a 1-meter sphere in 8 meters. For ions, this is a much shorter distance than either angular momentum limit. For electrons, the magnetic limit is a factor of two less than the space charge limit. Equation (7) provides insight into magnetically limited sheaths.

Although scattering can leave electrons trapped in the sheath, the number that are trapped cannot be substantial, unless they generate enough ions to balance their space charge. For large sheath dimensions, compared with the effective radius of the spacecraft, the surface charge depends only weakly on the sheath radius. Thus, while the magnetic field may modify particle trajectories and the sheath shape, the total number of electrons in the sheath is the same with or without the magnetic field.

Outside of the space charge sheath, the ionosphere is perturbed by weak electric fields that focus thermal current to the sheath edge and allow the plasma to satisfy the Bohm criterion at the sheath edge. How this is accomplished in a magnetoplasma is not known. For nonmagnetized plasmas, Parrot, *et al.* (1982) calculated self-consistent potentials and densities for the quasi-neutral presheath. Their analysis lead to a sheath edge potential of $0.7\theta_e$ and an incident current of $1.45\ j_{th}$. These results, modified to account for spacecraft motion, are used in the computer calculations.

ALGORITHMS

The analysis above describes the plasma surrounding a high-voltage spacecraft in terms of a nonneutral space charge sheath, a quasi-neutral presheath, and the undisturbed plasma. The potential variation in the presheath is small, less than θ_e. For spacecraft potentials of a hundred volts or more, this is beyond the accuracy of the calculations, and the potential variation in the presheath is ignored.

Throughout space, NASCAP/LEO and POLAR solve the variational form of Poisson's equation

$$\delta \iiint \left(\frac{\varepsilon_0}{2} |\nabla\phi|^2 + \rho\phi \right) dx = 0. \tag{8}$$

The variational form of Poisson's equation is used because it is easier to generalize to three dimensions and irregular zoning. A finite element approach is used to interpolate between nodal values of the potential. Equation (8) is solved using a scaled conjugate gradient algorithm.

The solution of Poisson's equation is straightforward; the complications stem from the determination of the space charge density. Outside of the sheath, the plasma is assumed to shield linearly,

$$\rho = -\frac{\varepsilon_0 \phi}{\lambda_D^2} \quad , \ |\phi| \leq 0.7\theta_e \ . \tag{9}$$

This approximation is used in both the NASCAP/LEO and POLAR codes. The use of an analytic approximation for the presheath charge density is necessary to prevent outrageous computing requirements. For a problem space of 10m x 10m x 10m, on the order of 10^9 macro particles would be needed to keep numerical fluctuations below the thermal energy of the particles.

Inside the sheath, NASCAP/LEO and POLAR use different algorithms to obtain the charge density. In POLAR, macro particles are tracked in from the sheath edge and their contribution to the space charge is accumulated in each element. The equations of motion of the macro particles include the Lorentz force, so the resultant charge density includes the effects of the earth's magnetic field. POLAR iterates calculations of space charge and potentials until convergence is obtained. This frequently requires days of computing on a desktop workstation. Some particles are neither collected by the object nor ejected from the sheath. These particles are followed for some number of bounces within the sheath. The number of bounces followed is the only free parameter in a POLAR calculation, and is usually chosen high enough (around 10) so that only a few percent of the particles are still bouncing at the end of the calculation.

NASCAP/LEO uses a simple, nonlinear analytical formula for the space charge. The function used is

$$\rho = -\frac{\varepsilon_0 \phi}{\lambda_D^2}\left(\frac{1+\frac{\phi}{\theta}C(\phi,E)}{1+\sqrt{4\pi\left|\frac{\phi}{\theta}\right|^3}}\right) \tag{10}$$

where the first factor represents the linear Debye screening from Equation (9), the numerator represents the density increase due to trajectory convergence, and the denominator reflects the density decrease due to particle acceleration in the sheath. Figure 1 shows equation 10 without the convergence factor. The convergence factor, $C(\phi,E)$, is a function of local field and potential.

$$C(\phi,E) = \left(\frac{\theta}{\phi}\right)\left(\frac{r_{sh}^2}{r^2} - 1\right) \tag{11}$$

where

$$\frac{r_{sh}^2}{r^2} = 2.29\left(\frac{E\lambda_D}{\theta}\right)^{1.262}\left(\left|\frac{\theta}{\phi}\right|\right)^{.509}. \tag{12}$$

The quantities r_{sh} and r refer to radii of an effective spherical diode. The numerical values were obtained by a fit to Langmuir-Blodgett spherical sheath results. C is zero for planar sheaths. When convergence is negligible, Equation (10) reduces, in the limit of large potentials, to the charge density of the accelerated plasma thermal current,

$$\rho = -\frac{j_{th}}{v}, \ \phi \to \infty. \tag{13}$$

Poisson's equation is solved iteratively with Equation (10) for the sheath potentials and fields. Equation (10) is not only used in NASCAP/LEO, but also in POLAR, where it is used to provide an initial potential estimate for the particle pushing iterations.

While the analytic charge density does not include magnetic field effects, it yields sheath potentials that lead to particle currents similar to particle currents computed from the self-consistent solutions. This occurs for two reasons. First, equation 7 says that the total charge in the sheath is the same whether there is a magnetic field or not. The formula in equation 10 distributes the charge incorrectly, but it gives the total charge correctly.

Second, the potential has about the right shape in the region from which particles are collected. Most of the particles collected are collected in less than a Larmor period, and thus their orbits are not perturbed that much by the magnetic field.

In both POLAR and NASCAP/LEO macro-particle trajectories are followed to determine how much current is collected by the spacecraft. The distribution of currents on the spacecraft is not particularly [illegible] perturbations near the sheath boundary lead to substant[illegible] code with quadratic interpolants for the potent[illegible] NASCAP will allow more accurate [illegible]

COMPARISON WITH FLIGHT DATA

Calculat[illegible] compared with flight data [illegible] 1989 Ma[illegible] discussion focuses on the [illegible] using particle tracke[illegible]

Space Power Experi[illegible] designed [illegible] spheres were biased up [illegible] experiment measured the steady state [illegible] rocket body. The rocket body po[illegible] achieve a balance between the electron current [illegible] secondary electron enhanced ion current to the rocket body. Both NASCAP/LEO and POLAR calculations show that current balance was achieved when the ion collecting sheath encompassed much of the electron sheath, reducing the electron currents. The calculations also show that the asymmetry introduced by the floating rocket body permitted almost all the electrons entering the sheath to be collected by the spheres; magnetic limiting played no role in the measured currents. Figure 2 shows calculated results from NASCAP/LEO for SPEAR I with one sphere biased to 46 kV and the spacecraft ground at -6 kV. Figure 2a shows potential contours and figure 2b shows the path of an electron in the potentials. Figure 3 shows a comparison between the measured and calculated currents. For SPEAR I, NASCAP/LEO and POLAR give almost identical results. The NASCAP/LEO calculations, using Equation (10) for the charge density, took a few CPU hours each. POLAR, pushing particles to obtain a self-consistent charge density, took a week of computer time for a single calculation.

The CHARGE-2 rocket consisted of a main payload section, containing an electron gun, and a smaller "daughter" payload section connected to the mainsection by a conducting tether. The NASCAP/LEO calculated ion currents collected were in agreement with the observed currents. The agreement extends to the currents to four small probes designed to measure sheath potentials. Electron collection by the mother had been first reported to exceed the magnetic limits of Parker and Murphy (Myers *et al.*, 1989). The Parker and Murphy limit was calculated using spherical probe whose surface area was equal to that of the rocket For data obtained above 250 km, NASCAP/LEO and POLAR calculations (Table 1), both agree with the measurements. The NASCAP/LEO and POLAR calculated currents are almost a factor of two greater than the spherical probe estimates. This demonstrates the importance of using the correct, 3-dimensional geometry. The POLAR self-consistent current is about 50 percent greater than the non-self-consistent NASCAP/LEO result. The POLAR calculation shows that sheath contraction, due to the effect of the magnetic field on electron trajectories, enables a larger number of electrons entering the sheath to be collected by the rocket body. Figure 4 shows the sheath contraction in the direction perpendicular to the magnetic field. Below 250 km, the measured currents were higher than either code predicts. Presumably, the higher currents were due to ionization of background gases by the electron beam.

CONCLUSION

Algorithms have been developed that calculate the plasma currents collected by high-voltage spacecraft in the ionosphere. The algorithms assume the plasma shields very small potentials ($|\phi| < 0.7\theta_e$) linearly on the Debye length scale. For larger potentials, NASCAP/LEO uses an analytic formula based on current continuity for the charge density. POLAR iterates particle-tracked densities and potentials until a self–consistent solution is achieved. Both codes push particles in from the sheath edge to determine how much current is collected. In both codes the full Lorentz force is used to determine the force on the particles. Comparisons with flight data from SPEAR I and CHARGE-2 show that the 3-dimensional geometry plays a major role in the determination of collected currents. The CHARGE-2 calculations show that self-consistent space charge increases the magnetic-limited electron currents.

Acknowledgement. This work was supported by Geophysics Laboratory, Hanscom Air Force Base, Massachusetts, under contract F19628-89-C-0032 and NASA Lewis Research Center, Cleveland, Ohio, under contract NAS3-23881.

REFERENCES

Beard, D. B., and F. S. Johnson, Ionospheric limitations on attainable satellite potential, *J. Geophys. Res.*, *66*, 4113, 1961.

Chen, F. F., Electric probes, in *Plasma Diagnostic Techniques*, edited by R. H. Huddleston and S. L. Leonard, pp. 113-200, Academic, Orlando, Fla., 1965.

Katz, I., G. A. Jongeward, V. A. Davis, M. J. Mandell, R. A. Kuharski, J. R. Lilley, Jr., W. J. Raitt, D. L. Cooke, R. B. Torbert, G. Larson, and D. Rau, Structure of the bipolar plasma sheath generated by SPEAR I, *J. Geophys. Res.*, *94*, 1450, 1989.

Laframboise, J. G., and J. Rubinstein, Theory of a cylindrical probe in a collisionless plasma, *Phys. Fluids*, *19*, 1900, 1976.

Laframboise, J. G., and M. Kamitsuma, The threshold temperature effect in high-voltage spacecraft charging, Proceedings of the Air Force Geophysics Laboratory Workshop on Natural Charging of Large Space Structures in Near Earth Polar Orbit: 14-15 September, 1982, AFGL-TR-83-0046, pp. 293-308. **ADA134894**

Langmuir, I., and K. B. Blodgett, Currents limited by space charge between concentric spheres, *Phys. Rev.*, *24*, 49, 1924.

Linson, L. M., Current-voltage characteristics of an electron emitting satellite in the ionosphere, *J. Geophys. Res.*, *74*, 2368, 1969.

Mandell, M. J., J. R. Lilley, Jr., I. Katz, T. Neubert, and N. B. Myers, Computer modeling of current collection by the CHARGE-2 mother payload, Submitted to *J. Geophys. Res.*, 1989.

Mott-Smith, H. M., and I. Langmuir, The theory of collectors in gaseous discharges, *Phys. Rev.*, *28*, 727, 1926.

Myers, N. B., W. J. Raitt, B. E. Gilchrist, P. M. Banks, T. Neubert, P. R. Williamson, and S. Sasaki, A comparison of current-voltage relationships of collectors in the earth's ionosphere with and without electron beam emission, *Geophys. Res. Lett.*, *16*, 365, 1989.

Neubert, T., M. J. Mandell, S. Sasaki, B. E. Gilchrist, P. M. Banks, P. R. Williamson, W. J. Raitt, N. B. Myers, K. I. Oyama, and I. Katz, The sheath structure around a negatively charged rocket payload, To be published in *J. Geophys. Res.*, 1989.

Parker, L. W., and B. L. Murphy, Potential buildup on electron-emitting ionospheric satellites, *J. Geophys. Res.*, *72*, 1631, 1967.

Parrot, M. J. M., L. R. O. Storey, L. W. Parker, and J. G. Laframboise, Theory of cylindrical and spherical Langmuir probes in the limit of vanishing Debye number, *Phys. Fluids*, *25*, 2388, 1982.

Rubinstein, J., and J. G. Laframboise, Upper bound current to a cylindrical plasma probe in a collisionless magnetoplasma, *Phys. Fluids*, *21*, 1655, 1978.

Rubinstein, J., and J. G. Laframboise, Theory of a spherical probe in a collisionless magnetoplasma, *Phys. Fluids*, *25*, 1174, 1982.

Rubinstein, J., and J. G. Laframboise, Theory of axially symmetric probes in a collisionless magnetoplasma: Aligned spheroids, finite cylinders, and disks, *Phys. Fluids*, *26*, 3624, 1983.

Winckler, J. R., The application of artificial electron beams to magnetospheric research, *Rev. Geophys.* *18*, 659, 1980.

Table 1. Calculated and measured collected current (mA)

Altitude [km]	Potential [volts]	Measured	Parker-Murphy[a]	NASCAP/LEO Collected Current	NASCAP/LEO Sheath Current	POLAR Self-consistent
165	390	35.8	0.4	0.6	8.3	-
168	150	6	0.3	0.5	3.5	-
232	475	20.4	5.5	7.9	42	-
251a	560	12.2	8.5	13.0	60	-
251b	440	14	8.1	12.9	52	-
256	440	15.6	8.6	13.5	54	-
260	440	18	8.9	14.1	55	22

[a]Calculated for sphere of radius 0.6 meters.

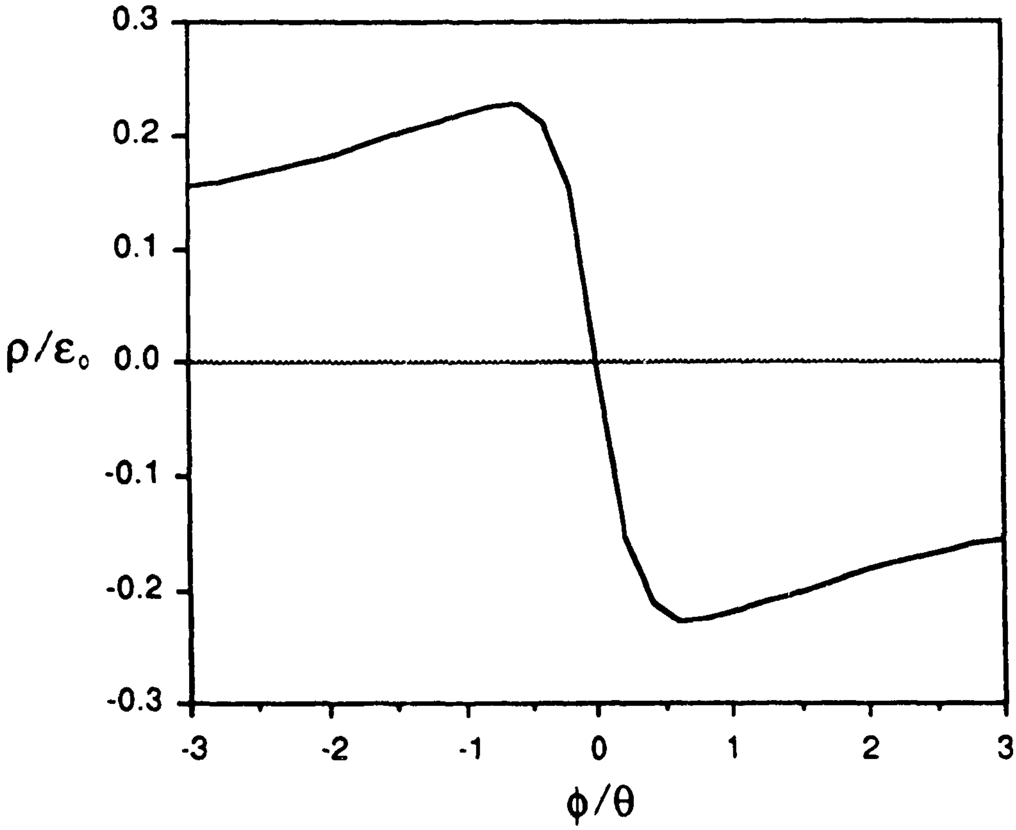
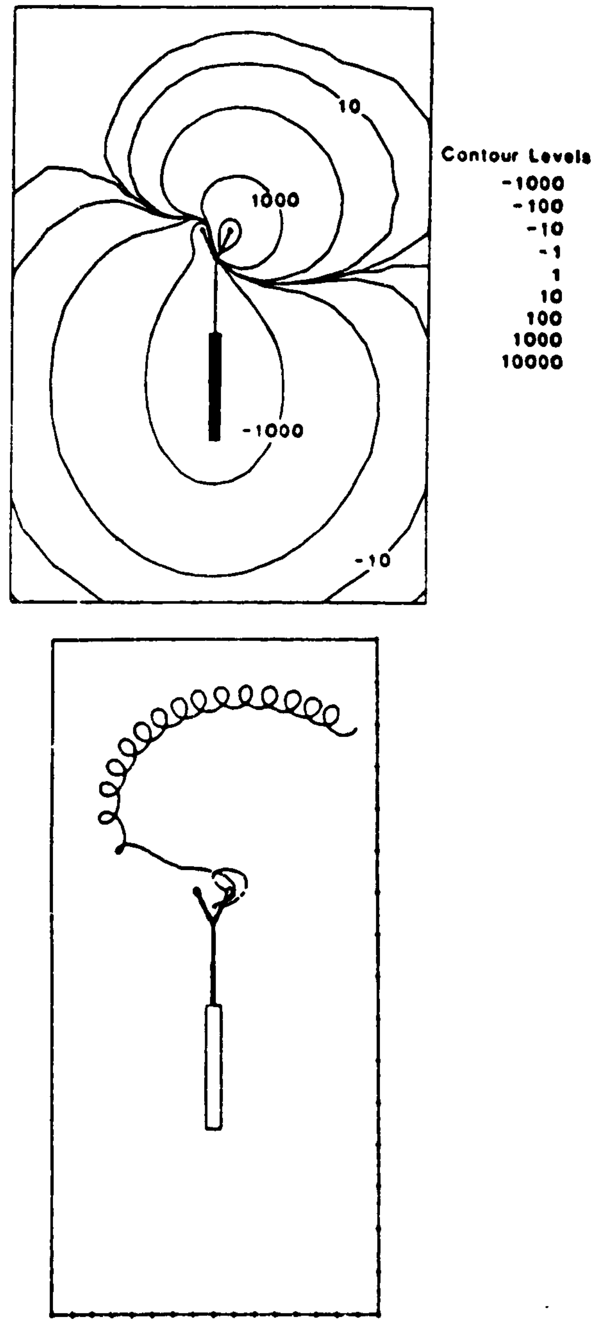


Fig. 1. Charge density as a function of potential from equation 10 for λ_D=1 m.



(a)

(b)

Fig. 2. (a) Potential contours calculated by NASCAP/LEO for the case with one sphere biased to 46 kV and the spacecraft ground at -6kV and (b) path of an electron in the potentials shown in (a). Note that the path is dramatically influenced by the presence of the ion-collecting sheath.

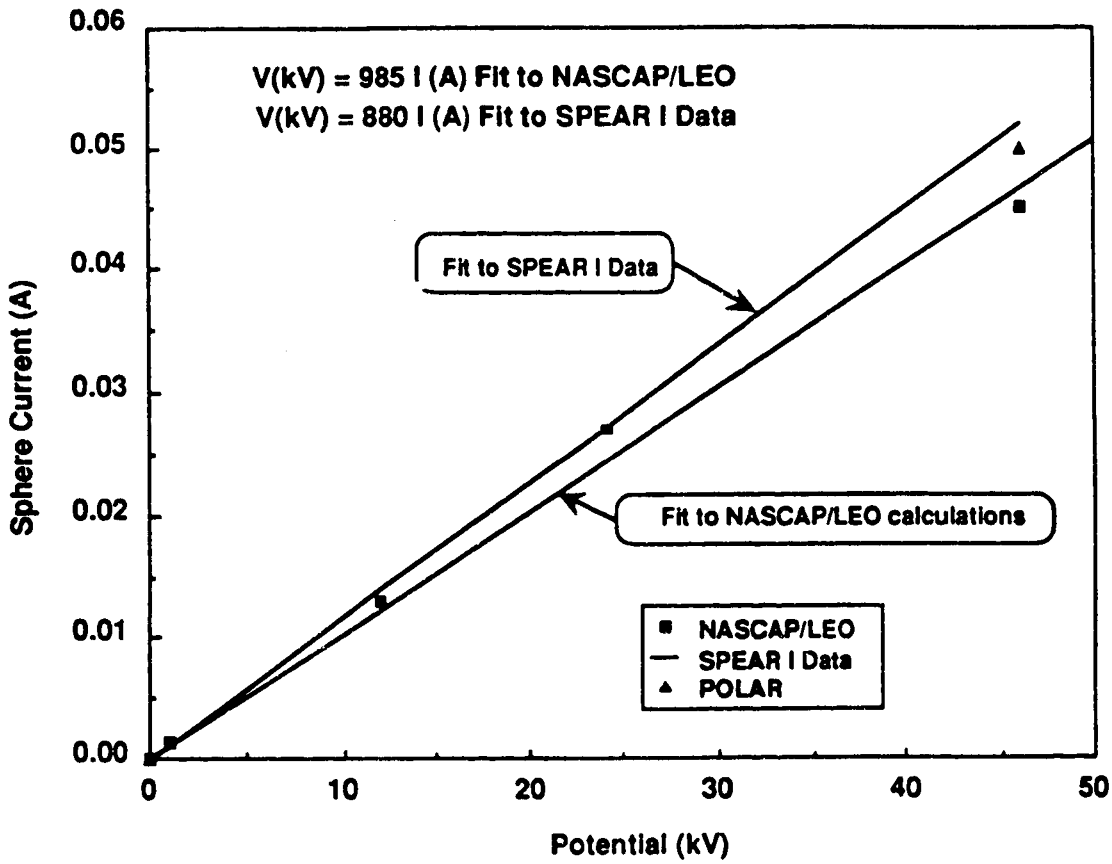
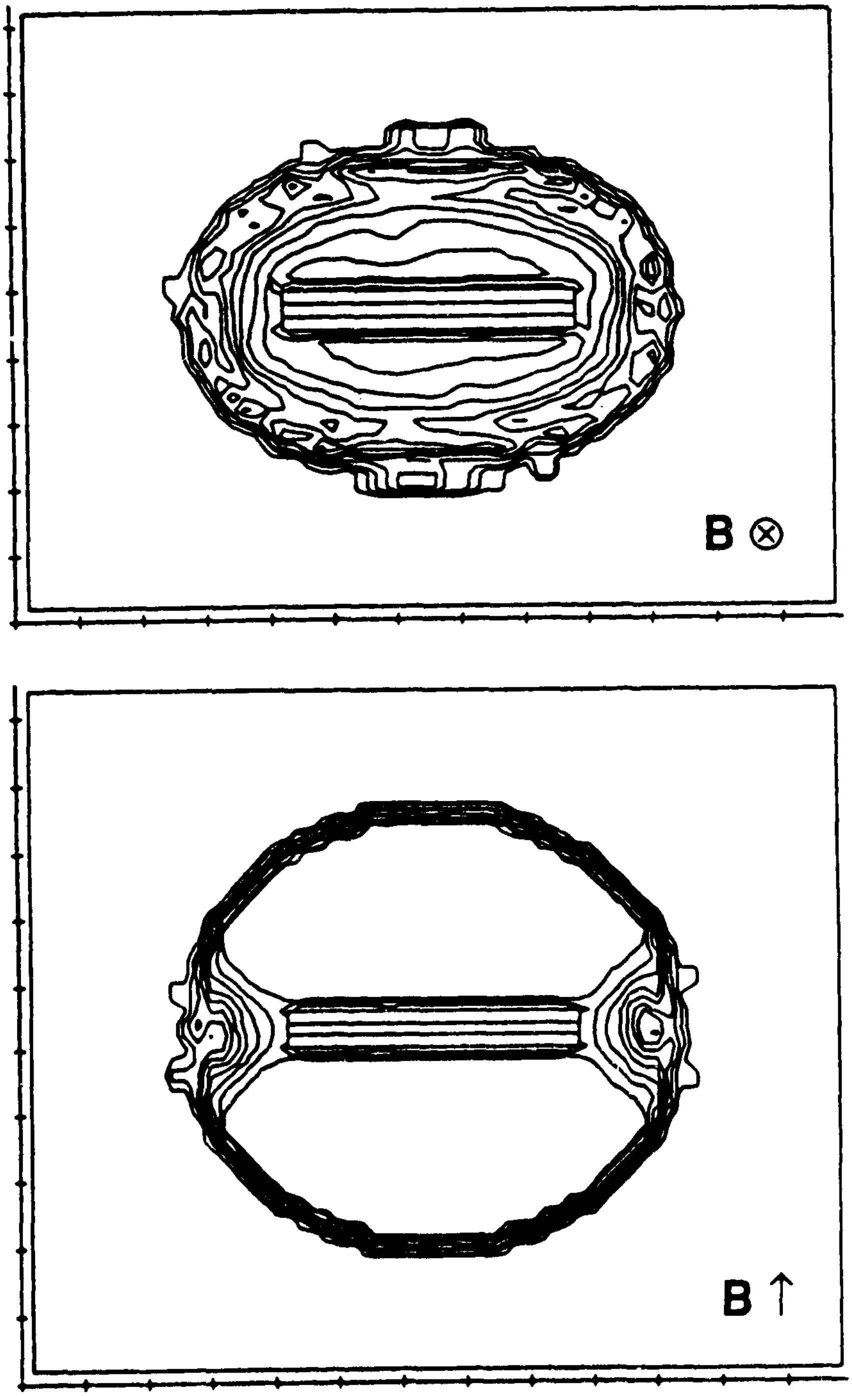


Fig. 3. Observed and calculated current collected by the 46 kV sphere as a function of applied potential.



(a)

(b)

Fig. 4. POLAR calculations of the sheath about the CHARGE-2 rocket perpendicular to and in the plane of the earth's magnetic field. Note how the sheath extends much farther away from the rocket in the direction of the earth's magnetic field.