Circuit Solver

This document describes SPIS electrical circuit solver.

SPIS basic spacecraft equivalent circuit results in the following matrix equation to be solved:

$$\frac{dV}{dt} = C^{-1}.\left(I_{plasma-net}(V) + R^{-1}.V\right) \quad (1)$$

where $I_{plasma\text{-}net}$ vector is the net plasma current (collected - emitted) on each SC surface element, the capacitance matrix C and the conductivity matrix R are straightforwardly derived from the equivalent circuit. In this equation, the (implicit) matrix and vector indices refer to *reduced electric nodes*. Three different types of electric nodes can be defined:

- super nodes: they are macroscopic and defined by the user through SPUI-UI (ESN-1, ESN-2, etc), they typically cover a sub-system of the spacecraft
- (regular) nodes: there is one per surface element in the spacecraft surface mesh; they are generated automatically by the code
- reduced nodes: when a set of different nodes (regular + super nodes) are connected by active biases, all but one of them drop out of the equations (1) since all potentials can be derived from one another, and the remaining node is referred to as the reduced node of the node set

Super nodes and regular nodes are described in the spacecraft equivalent circuit description page, but not the reduced nodes, since they are hidden for the user. At code level the spacecraft model deals with nodes and super nodes (and makes all the generation of nodes and reduced nodes), while the circuit model only knows reduced nodes.

The handling of the spacecraft absolute capacitance C_{sat} is yet not absolutely straightforward, since it may be connected in different ways to spacecraft electric super nodes. By default C_{sat} is distributed among all super nodes grounds proportionally to their surface area (hence as many capacitors between super node grounds and infinity as there are super nodes, with their total capacitance summing up to C_{sat}). This is a reasonable model of spacecraft absolute capacitance, which is in reality due to spacecraft sheath, hence more or less distributed uniformly all over the spacecraft. If he wishes so, the user may yet force the code to only plug C_{sat} between the spacecraft ground (node 0) and infinity. More on this in ../HowTo/Controlling NUM from UI.html#Csat and ../HowTo/Spacecraft circuit.html#Csat

The net current $I_{plasma\text{-}net}$ in (1) is supplied by the plasma solver at each spacecraft-plasma loop. Its time step is ruled by simulationDt parameter value and time step automatic determination rules.

In SPIS v3 the solver of equation (1) was a simple explicit solver with predetermined time steps. At each time step the right hand side of (1) is evaluated and V updated accordingly. After the update of the potential of each reduced node, the potential of all nodes (regular and super nodes) is computed from the biases.
NB: conversely, at the beginning of this procedure, the currents, which are collected on regular nodes, are summed over bias-connected nodes to obtained current on reduced nodes $I_{plasma\text{-}net}$ (of which vector indices are reduced nodes).

Since SPIS v4.0, a Newton-type implicit solver was implemented. The idea of Newton methods is that, if the intensity changes induced by potential changes can be anticipated by the solver thanks to a (crude) linear estimator (dI/dV), convergence can be sped up and fast

divergence in direction of fast evolutions (absolute charging here) can be avoided even for large time step.
The general scheme implemented in the new solver is that each current (collected or emitted) comes with its dI/dV matrix estimator (dI_i/dV_j matrix elements for surface nodes *i* and *j*) and a validity range (in potential). The linear estimator for the current change is used as long as the potential change is not larger than the validity range.
This new circuit solver can also take into account inductances. The resulting equation is somewhat changed and becomes of the second order. The detailed equations are given below.

Detailed equations of SPIS v4.0 RLC circuitry

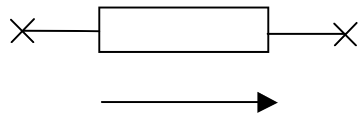
Symbols should be clear from electrical charts on the right hand side (*Js* are intensities between two nodes, while *Is* are total collected intensities on a node). Any node can be connected to others by any type of device (or not) and to plasma for emission and collection. In practice the user handles two types of nodes, regular nodes (the top of a surface mesh element) and electrical super nodes (the mass of a spacecraft sub system), but from an electrical circuit point of view this distinction does not matter since they are handled on the same footing.

Resistor

$$U_i - U_j = R_{ij} J^R_{ij},$$

or introducing the conductance $g = 1/R$:

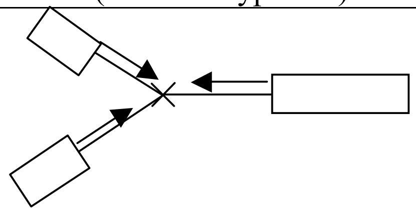
$$g_{ij}\left(U_i - U_j\right) = J^R_{ij}$$



Defining I^R_i as the total current arriving on node *i* due to resistors (Kirchhof type law)

$$I^R_i \equiv \sum_j J^R_{ji}$$

(with J^R_{ji} ï¿½= - J^R_{ij})



this can be rewritten under matrix form

$$\underline{I^R} = \underline{\underline{G}}.\underline{U}$$

where the elements G_{ij} of the conductance matrix $\underline{\underline{G}}$ ï¿½are not equal to the above g_{ij}, but simply derived from the above equations. A conductance g_{ij} indeed results in a contribution:

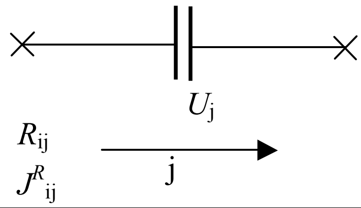
- $-g_{ij}$ to G_{ii} and G_{jj}
- $+g_{ij}$ to G_{ji} and G_{ij}

Capacitor

$$c_{ij}\left(U_i - U_j\right) = Q_{ij},$$

and by time derivation ($\dot{Q}_{ij} \equiv J^C_{ij}$):

$$c_{ij}\left(\dot{U}_i - \dot{U}_j\right) = J^C_{ij}$$



J^R_{j1i} J^R_{i0i}

Defining similarly $I^C{}_i$ as the total current arriving on node *i* due to capacitors $I_i^C \equiv \sum_j J_{ji}^C$ this can be rewritten under matrix form

$$\underline{I^C} = -\underline{\underline{C}}.\underline{\dot{U}}$$

where the elements C_{ij} of the conductance matrix $\underline{\underline{C}}$ ï¿½are derived from the above equations[1]. A capacitor of capacitance c_{ij} results in a contribution:

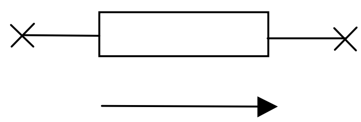
- $+c_{ij}$ to C_{ii} and C_{jj}
- $-c_{ij}$ to C_{ji} and C_{ij}

There is an extra contribution from spacecraft absolute capacitance C_{sat} (plugged between some SC ground[2] and "infinity"). From $C_{sat}\left(0 - \dot{U}_i\right) = J_{\infty i}^C$, describing the current from infinity to node *i* (the user defined ground to which C_{sat} is connected, typically SC body ground), we can derive the following contribution to the matrix $\underline{\underline{C}}$:

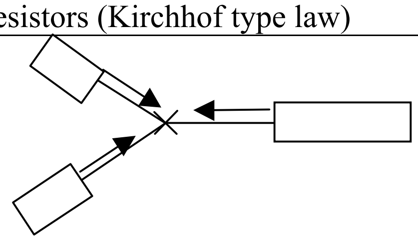
U_i L_{ij} U_j

- $+ C_{sat}$ to C_{ii}

The matrix $\underline{\underline{C}}$ ï¿½is symmetric. In absence of C_{sat}, if all nodes are interconnected by capacitors, a matrix $\underline{\underline{C}}$ ï¿½of dimension *N* has rank *N* ï¿½ 1 (and more generally *N ï¿½ n* if capacitor-related nodes form *n* connexes). In presence of C_{sat}, if all nodes are interconnected by capacitors, it has rank N (positive definite and invertible). In presence of of C_{sat}, and if all nodes are not interconnected by capacitors, many different situations may arise, in particular taking into account the fact that Csat can be plugged to one node or to many different ones (in which case C_{sat} is distributed between all C_{ii} for nodes *i* concerned).
It must also be noted that the values of the capacitances can be very variable. In particular C_{sat} is usually much smaller that coating capacitances (in GEO). It results in very different eigenvalues for the matrix $\underline{\underline{C}}$.

Inductance $$\left(U_i - U_j\right) = L_{ij}\,\dot{J}_{ij}^L,$$ or introducing the inverse of the inductance *k = 1/L*: $$k_{ij}\left(U_i - U_j\right) = \dot{J}_{ij}^L$$	

Defining $I^R{}_i$ as the total current arriving on node *i* due to resistors (Kirchhof type law)

$$I_i^R \equiv \sum_j J_{ji}^R$$ (with J_{ji}^R ï¿½= $-J_{ij}^R$)	

Again defining $I^L{}_i$ as the total current arriving on node *i* due to inductances $I_i^L \equiv \sum_j J_{ji}^L$ this can be rewritten under matrix form

$$\underline{\dot{I}^L} = \underline{\underline{K}}.\underline{U}$$

[1] Beware of the sign convention, different from the one used for resistors (this choice follows the one used internally in SPIS)

[2] See SPIS User Manuel to see how the user can specific to which node(s) C_{sat} can be plugged.

where the elements K_{ij} of the matrix of the inverse of the inductances $\underline{\underline{K}}$ ï¿½are derived from the above equations[3]. An inductance L_{ij} results in a contribution:

- $-1/L_{ij}$ to K_{ii} and K_{jj}
- $1/L_i$ to K_{ji} and K_{ij}

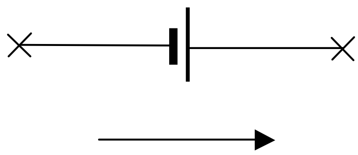
As we will see below it is indeed preferable to handle inductance on a different footing and use the state variables J^L_{ij}, the currents through the inductances. They can be obtained from the potentials by the equation

$\dot{J}^L_{ij} = k_{ij}(U_i - U_j)$ ï¿½(the equation above), orï¿½ï¿½ï¿½ï¿½ï¿½ï¿½ï¿½ï¿½

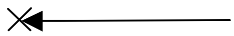
$\underline{\dot{J}^L} = \underline{\underline{H}}.\underline{U}$ ï¿½ï¿½ï¿½ï¿½in matrix form

hence with the following contributions to the rectangular matrix $\underline{\underline{H}}$:

- $k_{ij} = 1/L_{ij}$ to $H_{(ij)\,(i)}$
- $-k_{ij} = -1/L_{ij}$ to $H_{(ij)\,(j)}$

Bias (perfect source of potential) $$U_i - U_j = V_{ij}$$	

The current J^V_{ij} is arbitrary in this case. A simple way to model biases, implemented in SPIS, consist in eliminating one of the two connected nodes from the system. Currents to both nodes are summed and considered as currents to/from the remaining node, while the potential of the eliminated node (needed for current computation to/from it) is known from the remaining node potential from the relation above.

Arbitrary current law $$I^I_i = I\left(U_i, \{U_j\}_{j\neq i}, plasma, t\right)$$	

This law is very general. The current collected by a node can depend on its potential, other node potentials, the plasma, or explicitly on time, which can be a way to write a dependence on plasma. It covers as many different situations as:

- current source between two nodes, i and j: $I^I_i = - I^I_j = I_0$
- current source to plasma (ion/electron source), from node i: $I^I_i = I_0$
- arbitrary I(V) law between two nodes, i and j: $I^I_i = - I^I_j = I(U_i - U_j)$
- arbitrary collection or emission law from/to the plasma: $I^I_i = I\left(U_i, \{U_j\}_{j\neq i}, plasma, t\right)$

The latter case can be a simple particle collection, particle collection and reemission (secondary emission), or any emission law as e.g. the very steep Fowler-Nordheim (FN) law for field emission, behaving as an exponential of the inverse of the electric field. They can have an analytic dependence of variables of the system ($I^I_i = I(U_i - U_j)$), an analytic dependence of variables external to the electric circuit system (FN law dependent on normal

[3] With the same sign convention as for resistors here

electric field), or not analytic at all (collection from the plasma). We repeat that some of these laws can be very steep. For example the steep FN law can lead to discharges to start within 10^{11} s following a charging period that spread over minutes.

Finally adding up all currents to zero on each node (Kirchhoff's law) we get:

$$\underline{I}^C + \underline{I}^R + \underline{I}^L + \underline{I}^I = 0$$

Awkwardly deriving this equation and replacing each term gives the second order equation

$$-\underline{\underline{C}}.\underline{\ddot{U}} + \underline{\underline{G}}.\underline{\dot{U}} + \underline{\underline{K}}.\underline{U} + \underline{\dot{I}}^I = 0$$

It is preferable to introduce the currents in the inductances J^L_{ij} as new variables (in an arbitrary number) and keep first order differential equations:

$$-\underline{\underline{C}}.\underline{\dot{U}} + \underline{\underline{G}}.\underline{U} + \underline{\underline{P}}\underline{J}^L + \underline{I}^I = 0$$

$$\underline{\dot{J}}^L = \underline{\underline{H}}.\underline{U}$$

where $\underline{\underline{P}}$ ï¿½is the "projector" from J^L_{ij}'s onto I_i's: $P_{(i)\,(jk)} = -\delta_{ij} + \delta_{ik}$. The first order system can be written by blocks:

$$\begin{bmatrix} \underline{\dot{U}} \\ \underline{\dot{J}}^L \end{bmatrix} = \begin{bmatrix} \underline{\underline{C}}^{-1}.\underline{\underline{G}} & \underline{\underline{C}}^{-1}.\underline{\underline{P}} \\ \underline{\underline{H}} & 0 \end{bmatrix} . \begin{bmatrix} \underline{U} \\ \underline{J}^L \end{bmatrix} + \begin{bmatrix} \underline{\underline{C}}^{-1}.\underline{I}^I \\ 0 \end{bmatrix}$$